

Fugue No. 10

E minor

Well-Tempered Clavier Book I

Johann Sebastian Bach

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To read this essay in its hypermedia format, go to the Shockwave movie at <http://jan.ucc.nau.edu/~tas3/wtc/i10.html>.



Subject: Fugue No. 10, *Well-Tempered Clavier*, Book I

This is the only two-voice fugue in the *WTC*. Because a fugue requires at least two voices for its subject and answer relationship, the voicing is minimalistic. When it comes to counterpoint however this is one of Bach's most clever of fugues. In this analysis I'll demonstrate how it is:

- two fugues in one
- double counterpointed at the octave
- a canon
- like a Möbius strip
- desirous of a coda

Two Fugues in One

The timeline reveals this fugue to be highly symmetrical: four sections in an *abab* pattern. All of the music is encapsulated in the first nineteen measures. These contain an exposition and 1st development (line 1) followed by a re-exposition and 2nd development (line 2). Thus the first two lines comprise what I shall call *the fugue*.

What remains is double counterpoint of the fugue transposed to new keys. Containing no new music, line 3 and line 4 are derived from the fugue by double counterpoint. For this reason I have identified them as *the counterfugue*. So you can see that this fugue is really two fugues in one.

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Double Counterpointed at the Octave

Double counterpoint involves an exchange of registers; the melody that had been in the high voice is now heard in the low and vice versa. If it is helpful, you may think of double counterpoint like a game of leapfrog. At the beginning you leapt over the person in front of you. At the moment of your leap you were higher than him and he lower than you. Now it has come his turn to leap and the situation is reversed. This is what I meant by an *exchange of registers*.

The counterfugue is comprised entirely of double counterpoint at the octave. Bach has simply repeated the first nineteen measures with the high voice having moved to the low and vice versa.

In addition to its double counterpoint the counterfugue has also undergone transposition. Line 3 is a transposition of line 1 up a 4th. By contrast line 4 is a transposition of line 2 in the opposite direction, down a 4th.

If double counterpoint were new to you it would be instructive to compare analogous measures of the fugue and counterfugue. You can do this by holding the mouse down and hovering over corresponding measures in the timeline.

To find the corresponding measure in the counterfugue, add 19 to each measure in the fugue. So m. 11 of the fugue corresponds with m. 30 ($11+19=30$) of the counterfugue. Corresponding measures are vertically aligned and separated by one line, the even-numbered lines corresponding with each other, as do the odd.

A Canon

It is in the nature of a fugue to be imitative. But this fugue is more so than most. This is because it is also a canon. So that you may understand this relationship I should like to clarify the difference between a canon and a fugue.

The relationship between a fugue's subject and its answer is essentially one of imitation. In a *tonal answer* the contour is imitated with occasional interval mutation in order to maintain the same key. In a *real answer* the intervals and contour are identical, but transposed to a new key. In most fugues such strict imitation normally breaks down after the exposition.

Like the fugue, a canon generates one voice out of another by contrapuntal means. Whatever means is chosen (a different interval, rhythmic proportion, contrary motion, retrograde), the second voice is bound as if by law to continue imitating the first voice by the same means. This is what "canon" means: rule or law. The moment it breaks the rule, the episode is no longer canonic.

Remember this. A composition can be: (1) a canon and not a fugue, (2) a fugue and not a canon, or (3) both a canon and a fugue. Compositions in the third category are rare, therefore of special importance. This fugue belongs to that category; it is at the same time a canon and a fugue.

As a fugue this work employs a *real* relationship. The answer (low voice mm. 3-4) is a transposition of the subject (high voice mm. 1-2) from the key of em to bm. This little segment is a *canonic episode*; it does not qualify the entire work as a canon because the imitation is abandoned in m. 7.

Here is what makes this fugue a canon. Earlier I said that all of the music is contained in mm. 1-19. But there is another way to hear it. All of the music is

also contained in the high voice. Let us call lines 1-2 of the high voice Part A, and lines 3-4 part B. Everything to the Coda could be represented as follows:

Part A Part B (high voice)
Part B Part A (low voice)

So what I had called *the fugue* is nothing less than column one (above) and *the counterfugue* is column two. The columns illustrate how double counterpoint undergirds the canonic process; it is impossible to have a canon without it. So this fugue is a canon because it consists entirely of double counterpoint (Coda excluded).

Many of the canonic structures in this fugue have been animated. One voice gravitating to another indicates canon. Motions that are limited to the same lines indicate small-scale imitation (*canonic episodes*). Motions involving whole lines migrating to other lines indicate large-scale imitations as in: m. 1, m. 11, m. 20 and m. 30. These larger imitations are what make this fugue a canon.

Like a Möbius Strip

Now let's talk about lines. If you draw a line from Eisenach (Bach's birthplace) to Leipzig (where he died), you will come across Schulpforta, one of those delightful little Saxon towns with which Bach would have been very familiar. It was in Schulpforta that the mathematician August Möbius was born in 1790. Interestingly Möbius taught at Leipzig University where Bach had directed the *Collegium Musicum* 100 years before. Both men died in Leipzig.



While in Leipzig, Möbius answered a question on the geometric theory of polyhedra posed by the Paris Academy. His answer required the calculation of properties left unchanged by the continuous deformation of lines in space. His proof referenced what has since come to be known as the *Möbius strip*.

A Möbius strip is a surface that has only one side, like a sheet of paper with a front but no back. Here's how to make one. Cut a narrow slip of paper then write *FUGUE* on one surface and *COUNTERFUGUE* on the other. Tape the ends together with a 180-degree twist (as in the diagram bottom right).

Now draw a line paralleling the length of the strip. Take care not to lift your pencil from the surface. Keep drawing until the line has returned to its point of origin. Amazingly the line will have marked both "sides" of the strip. That you could do this without lifting your pencil proves that the Möbius strip really has one side and not two.

In the course of drawing the line you will have traversed through both the *FUGUE* and *COUNTERFUGUE*. The Möbius strip illustrates how these two are really made of the same music. It also illustrates the seamless manner in which Bach has moved from one to the other.

Measure 39 is very important to our understanding of this fugue. If you compare it with m. 1 you will hear that it is the point of return. It is like that point in the Möbius strip where the line returns to its point of origin. Continuation of the

process beyond m. 39 would have required another traverse of Bach's fugal Möbius strip. Thankfully he has interrupted the process and brought the fugue to a close by means of a four-measure coda.

The Möbius strip accomplishes its magic by means of its twist. The twist is what gives its plane a single side. Bach's fugue also has a twist that allows it to move seamlessly from fugue to counterfugue and back.

To appreciate the significance of Bach's twist it is important to emphasize that the fugue and counterfugue are analogous to each other in every way except for *one interval*. That interval is found in beat one of m. 10: a rising third between the 2nd and 3rd pitches. At the analogous point in m. 29 Bach has composed a rising fourth. It is a tiny difference but one that allows the counterfugue to return to the fugue in the correct key (m. 39). This return is abundantly clear in a comparison of the following pairs of measures.

m. 15 - m. 35
m. 16 - m. 36
m. 17 - m. 37

Desirous of a Coda

The significance of the interval deviation in m. 10 and m. 29 cannot be overstated. I call it, "the twist." You may have observed that the twist in the Möbius strip (bottom right) occurs in these measures.

The fugue leads naturally to *the counterfugue*. But without *the twist*, the counterfugue would not have returned to the fugue in its original key. Without the twist this work would have been a *modulating* or *spiral* canon. It would eventually have gotten to the right key but only after several more repetitions.

The twist turns this work instead into what is known as a *perpetual* canon--a closed and recursive system like that of the Möbius strip. Like the Möbius strip this fugue has a proclivity to repeat itself forever. Such a tendency is illustrated by the following comparison (DC stands for *Double Counterpoint*):

L1 is the DC of L3 *down* a 4th
L2 is the DC of L4 *up* a 4th
L3 is the DC of L1 *up* a 4th
L4 is the DC of L2 *down* a 4th

How is it all to end? I jest that the fugue is desirous of a coda. I rather think that the fugue would be quite content to repeat itself forever. In truth it is we who desire it to end. So by the insertion of a Coda Bach has brought it to a satisfying, and welcome, close.