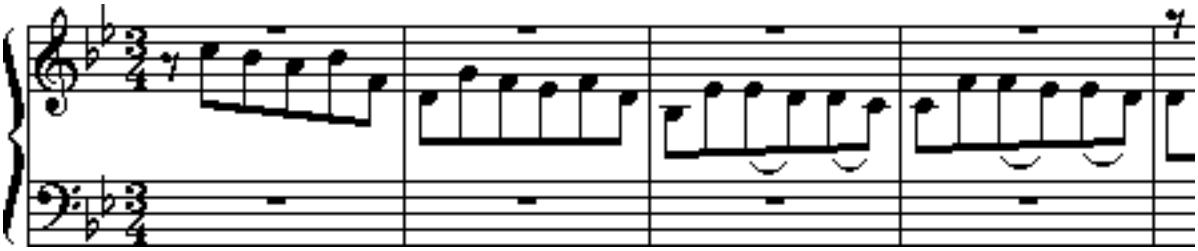
Fugue No. 21

B-Flat Major
Well-Tempered Clavier Book II
Johann Sebastian Bach

To read this essay in its hypermedia format, go to the Shockwave movie at http://bach.nau.edu/clavier/nature/fugues/Fugue45.html.



Subject: Fugue No. 21, *Well-Tempered Clavier*, Book II

- squares, chevrons, and pinwheels
- perceiving invariance
- brilliant or banal?
- permutation and triple counterpoint
- Pluto, Plato, and Lewis

Squares, Chevrons, and Pinwheels

Explore the tricolored square to the right! Each quadrant can be dragged to occupy the place of others. Try it! As you experiment, ask yourself: How many variations can I create? What shapes do they produce? Do some shapes repeat with color variation? Are some variations without shape? How many times can I repeat a process without duplicating a pattern? Before reading further, I encourage you to play with that square for as long as it interests you.

So, did you discover the shapes: square, chevron, and pinwheel? Every action yields one of these. Did you notice how the chevrons point right, left, up, and down; every pinwheel is poised to spin clockwise or counter clockwise; and the squares are dark on light or vice versa? You ask, what's so great about that? It seems natural enough. But this sort of thing doesn't happen by accident. The magic square's ability to do this is evidence of design.

An invariable arrangement would have been incapable of producing anything new (what musicians call *development*). Try this arrangement and you'll see what

I mean. It is not that these are ugly; they are actually quite attractive though totally incapable of variation — at least by the process of swapping quadrants. They are 100% *invariant*.

The difference between variable and invariable arrangements is one of orientation. In the former, each quadrant has been rotated by 90 degrees. In the latter, each quadrant has the same orientation.

But there is something even more productive about rotation. Each quadrant employs the same elemental pattern, one that does not automatically disclose its potential for the creation of squares, chevrons, or pinwheels. Only when combined with itself, in fourfold rotation, can the pattern produce shapes not found within itself.

Perceiving Invariance

If these designs are variations, how can the process that made them be said to involve invariance? Well, each transformation produces something *new*, while retaining something *old*. The squares, chevrons, and pinwheels are invariant, while their color schemes and alternating directions are variable.

The generation of new ideas from old lies at the very center of fugal technique. It is all about the process (arguably more so than form). The process requires a pattern with potential for transformation, and not just any pattern will do! Only asymmetric patterns are capable of transformation, and they must be properly arranged. While this particular pattern is potentially transformable, without rotation it cannot yield variation.

Some patterns cannot be changed by *any* process. The more symmetrical their kernel shape, the less variable they are. Still others yield variations so abstract as to be difficult to remember. Cognition is important here. If the brain can't perceive a pattern, then neither can it perceive its variation. In art, patterns that are impossible to perceive will produce transformations that, for all intents and purposes, do not exist. If it takes a computer to detect, then variation has not (cognition-wise) occurred. The trick is to find patterns and processes that balance variation with invariance — both of which can be perceived and remembered.

Brilliant or Banal?

A brilliant fugue employs variation with the right dose of invariance. If totally invariant, it will not produce new stuff from old and can't be developed. If too abstract, its variations may be impossible to remember. Bach's genius is in the knitting together of ideas that are (a) memorable, (b) capable of variation, while (c) retaining a balance of new and old. We often call this the principle of unity in variety. Bach called it *invention.*

Before writing an exposition, or developing anything, Bach worked out a fugal complex, a bit of counterpoint establishing the fugue's precompositional possibilities — what variations it could produce, and the processes that would produce them. He would then apply those processes to replicate the complex with variation. In addition to variation of texture and keys, Bach routinely indulged in

mode switching; this fugue is in B-flat major, but lends its subject briefly to g minor and c minor.

We have a word for the banal works that can't produce new from old. We call them "book fugues." These are the efforts of beginners, where the voices enter one by one while the audience leaves two by two (that's an old joke about the bad rap fugues get because of so many bad ones about). When assigned to write a fugue, composition students often emulate the form but forget about the process. They'll write a subject, present it in an exposition, and develop it (usually limited to modulations, a stretto, and a couple of sequences) then be done.

Bach's fugues are *not* book fugues. When it comes to this particular form, it is no exaggeration to say that Bach wrote the book. Everybody since has been trying to understand why he could write such beautiful music while conforming to such a confining form. The answer is that, while the form may be confining, its processes are anything but. Bach's secret weapons were invention and process — techniques that allowed his ideas to regenerate with unity in variety.

Permutation and Triple Counterpoint

One process that will insure complete variation without repetition is *permutation.* If followed to its end, permutation will guarantee that all variations have happened. Here, for example, is the process for finding every permutation of the magic square: drag the same square 11 times clockwise, then drag it diagonally once, then repeat step one. This yields a total of 24 variations.

Have you found the permutation pattern of this fugue?[2] If not, and you'd like to discover it for yourself, don't click the next sentence.

Reveal the permutation!

Notice how each column of the animation represents successive statements of the fugal complex. Pair them in your mind's eye. Are they all used?

Ah, I can hear your objection now: Bach didn't use the last one! What about that? The implication of your question seems to be that a process, once started, must be maintained to its conclusion. If not, then there must be a defect either in the composer's imagination or in the fugue's form. If imaginatively defective, then Bach ignorantly missed an opportunity for variation. If formally defective, then the fugue suffers for want of its unrealized potential.

In his book on the *Art of Fugue,* Donald Tovey observes that Bach never used every permutation.[3] There is an obvious reason for this — length! Not so obvious are the many musical reasons why Bach rejected various possibilities. One musical reason involves texture; the span between voices may have been too

[2] This work and its B-flat Major counterpart in Book I are both permutation fugues. Both have a subject and two countersubjects in triple counterpoint involving a predictable pattern of rotation. In the earlier work I related the permutation to the pattern of an Amish quilt.

[3] Donald Francis Tovey, *A Companion to The Art of Fugue* (London: Oxford University Press, 1931), 70-71, cited in Laurence Dreyfus, *Bach and the Patterns of Invention* (Cambridge: Harvard University Press, 1996), 185.

wide, or dense.

Another possibility involves invertible counterpoint. Also called *contrapuntal inversion,* this is a technique where two or more voices swap melodies — just like the magic square. By now you know this fugue well enough to recognize that mm. 32-84 are all about invertible counterpoint. Because three voices are involved we call it *triple counterpoint.*

Unlike the magic square, melodies can be inverted in many ways, with the most common intervals being the octave (8va), 10th, and 12th. Without delving into details, here's what you need to know. Three voices in triple counterpoint have but six textural permutations. But with some of these textures having possibility for inversion at *more than one interval,* potential variations actually number in the hundreds! So you can see that Bach actually rejected vastly more than he accepted.

Which brings me to the most important point. The person who made the fugue was a thoughtful and reflective human being, not a machine. He felt no obligation to follow a process to idiotic conclusions. He had the ability to evaluate aesthetic worth. He made choices, discarding the flawed so that we might enjoy the perfect. Far from being a defect, the rejection of potential variations reveals Bach's editorial skills, another reason why his fugues have survived where others have not.

So, what exactly did Bach bring us? What did he consider to be the most satisfying? With mm. 32-35 as the model, mm. 40-43 employ invertible counterpoint at the 10th and 12th. This is followed by mutation to minor and invertible counterpoint at the 8va and 12th in mm. 47-50. Retaining that chunk, inverted at the 8va and returned to major, and you get mm. 54-57. Now grab the original complex, invert it at the 8va in minor, and you'll hear mm. 63-66.[4] The complex of mm. 78-84 continues the preceding texture, followed by return to where it all began.

Pluto, Plato, and Lewis

> The human mind, overcome by the body's blindness, cannot discern by its dim light the delicate connections between things. But why does the mind burn with such desire to discover the hidden aspects of truth? Does it know what it is so eager to know?
>
> Boethius: Poem 3, Book V
> *The Consolation of Philosophy*[5]

In his "Organization of the Fugue" (from *Der Freie Satz)* Heinrich Schenker

[4] Each of the foregoing also involves rotation of motives between voices. The essential counterpoint is worked out in David Ledbetter, *Bach's Well-Tempered Clavier: the 48 Preludes and Fugues* (New Haven: Yale University Press, 2002), 321.

[5] Boethius, *Consolatio Philosophiae,* trans. Richard H. Green (Mineola: Dover Publications, 2002), 97.

wondered of “the way in which a composer arrives at his initial inspiration,” describing the power of invention as “one of the unfathomable secrets of creativity...a veritable clairvoyance which envisions a more distant level before the nearer one is clear in the consciousness.” Schenker continues: “Only a genius can command such far-reaching forward and backward perception.”

To those who think of the fugue as a type of creation, Schenker’s words are revelatory: “Just as this fugue is unique, so is the law which is the law of its life equally unique: the fugue itself gave rise to this law, not Bach — with the power of genius he merely recognized it and submitted to it.”

The ability to recognize what is bigger than us, and submit to it, has many parallels. One hundred years ago the astronomer Percival Lowell (working in my hometown of Flagstaff, Arizona) observed irregularities in the orbits of Uranus and Neptune. These deviations could be explained only by the influence of a trans-Neptunian object of a certain size and orbit. Although Lowell would never see “Planet-X,” as he called it, he believed in its existence long before Clyde Tombaugh made the official discovery.

Lowell was wrong about some things, like his theory about canals on Mars. But about Pluto he was on target. He was able (paraphrasing Schenker) to envision a distant level in intimate connection with nearer ones, not yet clear in the consciousness. We’re talking about the ability to recognize truth where evidence is minimal or nonexistent. For the sake of this argument, let us think of Truth and Beauty as being, if not one and the same, at least intimately connected.

Aristotle dealt with the problem this way. Truth (Beauty) is deduced from first principles, which cannot be deduced. Were a principle to be deducible then it could not be first. Because first principles are the basis for Truth, and because they cannot be deduced, Truth itself is not the product of reason. While true statements can be propositionalized, these are corollaries of the Truth.

In other words, Truth (Beauty) is bigger than us and innately realized. In certain cases we know Truth because we were born with the ability to recognize it. We were hardwired! In other cases we know Truth because it was given to us. We were told of it and believed.

In his *Dialogues,* Plato posed a problem to Socrates having to do with the order in which we realize the truth of a thing. Is something true because I *know* it to be true, or do I know it to be true because it *is* true?

Relating Plato to Pluto, was the truth of Pluto’s existence presaged by Lowell’s intuition and Tombaugh’s proof, or were these the products of that truth? Now the answer to this particular problem is obvious. Of course Pluto truly exists regardless of Lowell’s or Tombaugh’s intuitions or proofs. But sometimes these distinctions are not so easy to make. Consider a different example.

Suppose you are taking a multiple-choice test. You compare the various options and select the right one. But how did you know which one was true without first knowing which were false? And how did you know which were false without knowing which was true? Which came first?

To put this argument on a practical plane, How did Bach know that the permutations of this fugue were the most beautiful? How do we know that he retained the best and rejected the rest? Is it possible to propositionalize art so as

to prove its beauty? If so, upon what principle is the proposition framed? Thus framed, does not Truth underlie Bach's reasoning, and our apprehension of it? And if Truth, how did he (and how do we) know it to be true?

Responding to Plato's question, Socrates recounted how his servant, a boy named Meno, realized the solution to a geometrical problem even though he had no training in geometry. Socrates drew a square and asked Meno to create one that was twice as large.

Meno responded by doubling the length of each side. Realizing that he had quadrupled the original, Meno then divided each square in half. "Plato's square" (as this figure has come to be known) is significant for two reasons. First, it is the product of syllogism, the middle term of a ratio.

if ax4=b
and b/2=c
then c/2=a

But Socrates was less interested in propositional truth (the product of the syllogism) than he was in the Truth behind the syllogism itself. How could Meno, unschooled in geometry, have known the truth of his solution? Socrates concluded that Meno had remembered something with which he had been born. Essentially Socrates was expressing the truth of Aristotle's principle of the irreducibility of first principles. Meno "just knew" the truth of his solution. He was born with the ability to see that truth.

The great English writer, C. S. Lewis proposed: "We do not want merely to *see* beauty, though, God knows, even that is bounty enough. We want something else which can hardly be put into words — to be united with the beauty we see, to pass into it, to receive it into ourselves, to bathe in it, to become part of it."

But what is this Beauty, this Truth, of which we long to be a part? How do we know that it is beautiful? How do we know that it is true? Desire of Truth is, I believe, the basis for all art. Again I quote Lewis who wrote that beauty is, "the scent of a flower we have not found, the echo of a tune we have not heard, news from a country we have never yet visited." In search of such a beauty Bach wrote this fugue, rejecting its less desirable possibilities, retaining the best.

Unlike most people today, who believe that we create our desires or that they are environmentally conditioned, C. S. Lewis believed that desire is innate, thereby signifying the realness of its object. One cannot long for that which does not exist. Like Schenker's clairvoyance that envisions a more distant level before the nearer one is clear in the consciousness, Lewis believed that our desires are proof of that which they desire. We have hunger; there is food. We desire companionship; there are friends. We need rest; there is sleep.

Lewis continues, "The sweetest thing in all my life has been the longing...to find the place where all the beauty came from." Longing itself, Lewis reasoned, is the proof. This longing is sweet because it manifests the reality of beauty. We long for beauty because it is outside of ourselves, something of which we aspire to be

part. The artist is a subcreator of beauty that exists in the mind of the Creator.[6]

To use another analogy, a mirror cannot emit light. A mirror reflects light emanating from another source. Artists and their creations are the mirrors; beauty is the candle. There is an English idiom that expresses the essence of my argument: "You'll know it when you see it." This saying is usually said when one is unable to express qualities distinguishing one thing from another. Yet it radiates confidence that others, too, will recognize that "thing" without even having seen it, or even what it is. It asserts the realness of the object of desire — that which is hoped for but not yet seen.

[6] The term *subcreator* is from J. R. R. Tolkien's essay "On Fairy-Stories." Tolkien writes that the goal of fantasy is to give the reader "a sudden glimpse of the underlying reality or truth." In fantasy, authors are subcreators who "make...because we are made: and not only made, but made in the image and likeness of a Maker." Similarly, Luther taught that only God creates, with humans assembling and arranging that creation.