Robust Channel Estimation for OFDM Systems with Rapid Dispersive Fading Channels

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Abstract—Orthogonal frequency-division multiplexing (OFDM) modulation is a promising technique for achieving the high bit rates required for a wireless multimedia service. Without channel estimation and tracking, OFDM systems have to use differential phase-shift keying (DPSK), which has a 3-dB signal-to-noise ratio (SNR) loss compared with coherent phase-shift keying (PSK). To improve the performance of OFDM systems by using coherent PSK, we investigate robust channel estimation for OFDM systems. We derive a minimum mean-square-error (MMSE) channel estimator, which makes full use of the time- and frequency-domain correlations of the frequency response of time-varying dispersive fading channels. Since the channel statistics are usually unknown, we also analyze the mismatch of the estimator-to-channel statistics and propose a robust channel estimator that is insensitive to the channel statistics. The robust channel estimator can significantly improve the performance of OFDM systems in a rapid dispersive fading channel.

Index Terms—Diversity combining, orthogonal frequency-division multiplexing, robust channel estimation, time-varying dispersive fading.

I. INTRODUCTION

MULTIMEDIA wireless services require high-bit-rate transmission over mobile radio channels. To reduce the effect of intersymbol interference (ISI) caused by the dispersive Rayleigh-fading environment [1], the symbol duration must be much larger than the channel delay spread. In orthogonal frequency-division multiplexing (OFDM) [2]–[9], the entire channel is divided into many narrow subchannels, which are transmitted in parallel, thereby increasing the symbol duration and reducing the ISI. Therefore, OFDM is an effective technique for combating multipath fading and for high-bit-rate transmission over mobile wireless channels.

To eliminate the need for channel estimation and tracking, differential demodulation can be used in OFDM systems, at the expense of a 3–4-dB loss in signal-to-noise ratio (SNR) compared with coherent demodulation. Accurate channel estimation [5], [6], [8] can be used in OFDM systems to improve their performance by allowing for coherent demodulation. Furthermore, for systems with receiver diversity, optimum combining can be obtained by means of channel estimators. In [5], [6], and [8], a channel estimator for OFDM systems has been proposed based on the singular-value decomposition or frequency-domain filtering. Time-domain filtering has been proposed in [8] to further improve the channel estimator performance. However, the best time- or frequency-domain filtering shapes for channel estimation has not been studied.

In this paper we investigate minimum mean-square-error (MMSE) channel estimation for OFDM systems. We first derive the MMSE estimator, which makes full use of the correlation of the channel frequency response at different times and frequencies. In particular, for mobile wireless channels, the correlation of the channel frequency response at different times and frequencies can be separated into the multiplication of the time- and frequency-domain correlation functions. Hence, our MMSE channel estimator can be a frequency-domain filter using the fast Fourier transform (FFT), followed by time-domain filters. Since the channel statistics, which depend on the particular environment, are usually unknown, we present a robust estimator, that is, an estimator that is not sensitive to the channel statistics. Computer simulation demonstrates that the performance of OFDM systems using coherent demodulation based on our channel estimator can be significantly improved.

The paper is organized as follows. Section II introduces the statistics of the mobile wireless channel and briefly describes the OFDM system with channel estimation. Then, Section III derives the basic MMSE channel estimator for the OFDM system. Next, Section IV presents a robust channel estimator design approach. Finally, Section V presents computer simulation results to demonstrate the effectiveness of this channel estimator for OFDM systems in rapid dispersive fading channels.

II. CHANNEL STATISTICS AND OFDM SYSTEMS

Before investigating channel estimation for OFDM systems in mobile radio channels, we briefly describe the channel statistics, emphasizing the separation property of mobile wireless channels, which is crucial for simplifying our MMSE channel estimator. In this section we also briefly describe an OFDM system with receiver diversity.

A. Statistics of Mobile Radio Channels

The complex baseband representation [10] of a mobile wireless channel impulse response can be described by

\[ h(t, \tau) = \sum_k \gamma_k(t) \delta(\tau - \tau_k) \]

(1)

where \( \tau_k \) is the delay of the \( k \)th path and \( \gamma_k(t) \) is the corresponding complex amplitude. Due to the motion of the
vehicle, $\gamma_k(t)$'s are wide-sense stationary (WSS) narrowband complex Gaussian processes, which are independent for different paths.

We assume that $\gamma_k(t)$ has the same normalized correlation function $\rho_k(\Delta t)$ for all $k$, and, therefore, the same normalized power spectrum $p_k(\Omega)$. Hence

$$ r_{\gamma k}(\Delta t) \triangleq E\{\gamma_k(t + \Delta t)\gamma_k^*(t)\} = \sigma^2_{\gamma k}(\Delta t) $$

(2)

where $\sigma^2_{\gamma k}$ is the average power of the $k$th path.

Using (1), the frequency response of the time-varying radio channel at time $t$ is

$$ H(t, f) \triangleq \int_{-\infty}^{\infty} h(t, \tau)e^{-j2\pi f\tau}d\tau = \sum_k \gamma_k(t)e^{-j2\pi f\tau_k}. $$

(3)

Hence, the correlation function of the frequency response for different times and frequencies is

$$ r_H(\Delta t, \Delta f) \triangleq E\{H(t + \Delta t, f + \Delta f)H^*(t, f)\} = \sum_k r_{\gamma k}(\Delta t)e^{-j2\pi \Delta f\tau_k}. $$

$$ = r_{\gamma}(\Delta t)\left(\sum_k \sigma^2_{\gamma k}e^{-j2\pi \Delta f\tau_k}\right) $$

$$ = \sigma^2_{\gamma}r_{\gamma}(\Delta t)r_f(\Delta f) $$

(4)

where $\sigma^2_{\gamma}$ is the total average power of the channel impulse response defined as

$$ \sigma^2_{\gamma} \triangleq \sum_k \sigma^2_{\gamma k} $$

(5)

$$ r_f(\Delta f) = \sum_k \sigma^2_{\gamma k}e^{-j2\pi \Delta f\tau_k}, $$

(6)

It is obvious that $r_{\gamma}(0) = r_f(0) = 1$. Without loss of generality, we also assume that $\sigma^2_{\gamma}$ is 1, which, therefore, can be omitted from (4).

From (4), the correlation function of $H(t, f)$ can be separated into the multiplication of a time-domain correlation $r_{\gamma}(\Delta t)$ and a frequency-domain correlation $r_f(\Delta f)$. $r_{\gamma}(\Delta t)$ is dependent on the vehicle speed or, equivalently, the Doppler frequency, while $r_f(\Delta f)$ depends on the multipath delay spread. With the separation property, we are able to simplify our MMSE channel estimator described in the next section.

For an OFDM system with block length $T_f$ and tone spacing (subchannel spacing) $\Delta f$, the correlation function for different blocks and tones can be written as

$$ r_H[n, k] = r_{\gamma}[n]r_f[k] $$

(7)

where

$$ r_{\gamma}[n] \triangleq r_{\gamma}(nT_f) \quad r_f[k] \triangleq r_f(k\Delta f). $$

(8)

From Jakes’ model [11]

$$ r_{\gamma}[n] = J_0(n\omega_d) \triangleq r_f[n]\sigma^2_{\gamma} $$

(9)

where $J_0(x)$ is the zeroth-order Bessel function of the first kind, and its Fourier transform (FT) is

$$ p_f(\omega) = \begin{cases} 2\omega_d \sqrt{1 - (\omega/\omega_d)^2}, & \text{if } |\omega| < \omega_d \\ 0, & \text{otherwise} \end{cases} $$

(10)

In the above expression $\omega_d = 2\pi f_d T_f$ and $f_d$ is the Doppler frequency, which is related to the vehicle speed $v$ and the carrier frequency $f_c$ by

$$ f_d = \frac{v f_c}{c} $$

(11)

where $c$ is the speed of light. For example, for a system with carrier frequency $f_c = 2$ GHz, $f_d = 184$ Hz when the user is moving at 60 mi/h.

B. OFDM Systems with Channel Estimator

The OFDM system with channel estimation considered in this paper is shown in Fig. 1. The Reed–Solomon (RS) code across tones is utilized in the system to correct the burst errors resulting from frequency-selective fading. Since the phase of each subchannel can be obtained by the channel estimator, coherent phase-shift keying (PSK) modulation is used here to enhance the system performance.

For a diversity receiver, the signal from the $m$th antenna at the $k$th tone and the $n$th block can be expressed as

$$ x_m[n, k] = H_m[n, k]a[n, k] + w_m[n, k]. $$

(12)

In the above expression $w_m[n, k]$ is additive Gaussian noise from the $m$th antenna at the $k$th tone and the $n$th block, with zero-mean and variance $\rho$. We also assume $w_m[n, k]$ is independent for different $n$'s, $k$'s, or $m$'s. $H_m[n, k]$, the frequency response at the $k$th tone and the $n$th block corresponding to the $m$th antenna, is assumed independent for different $m$'s, but with the same statistics. $a[n, k]$ is the signal modulating the $k$th tone during the $n$th block and is assumed to have unit variance and be independent for different $k$'s and $n$'s.

With knowledge of the channel parameters, $a[n, k]$ can be estimated as $a[n, k]$ by an MMSE combiner

$$ y[n, k] = \frac{1}{\sum_{m=1}^{P}|H_m[n, k]|^2} \sum_{m=1}^{P} H_m[n, k] x_m[n, k]. $$

(13)

However, the multipath channel parameters are time varying.
and are usually unknown. Hence, a channel estimation algorithm must be derived to obtain accurate estimation of the channel parameters.

Since the channel corresponding to each antenna has the same statistics, the channel estimator for each antenna has the same coefficients. Furthermore, the estimator for each antenna works independently since the signal from the other antennas carries no information about the channel parameters corresponding to this antenna. Therefore, the subscript $m$ is eliminated from $H_m[n,k]$ in the next two sections.

### III. MMSE Channel Estimation

If the reference generator in Fig. 1 can generate an ideal reference $a[n,k]$, then a temporal estimation of $H[n,k]$ can be obtained as

$$\hat{H}[n,k] = x[n,k]a^*[n,k] = H[n,k] + w[n,k]a^*[n,k]$$

where the superscript $*$ denotes the complex conjugate. As indicated in the previous section, $H[n,k]$'s for different $n$'s and $k$'s are correlated; therefore, an MMSE channel estimator can be constructed as follows:

$$\hat{H}[n,k] = \sum_{m=-\infty}^{0} \sum_{l=-\infty}^{K-k-1} c[m,l,k] \hat{H}[n-m,k-l]$$

where $c[m,l,k]$'s are selected to minimize

$$\text{MSE}(\{c[m,l,k]\}) = E[\hat{H}[n,k] - H[n,k]]^2.$$ 

(16)

$K$ in the above expression is the number of tones in each OFDM block.

Denote

$$c[m,k] \triangleq \begin{pmatrix} c[m,k-1,k] \\ \vdots \\ c[m,0,k] \\ \vdots \\ c[m,-K+k,k] \end{pmatrix}$$

(17)

$$c(\omega;k) \triangleq \begin{pmatrix} c(\omega;1) \\ \vdots \\ c(\omega;K) \end{pmatrix}$$

(18)

$$C(\omega) \triangleq (c(\omega;1), c(\omega;2), \ldots, c(\omega;K)).$$

(19)

Then, using the separation property (7), it can be shown (see the Appendix) that the estimator coefficients are given by

$$C(\omega) = U^H \Phi(\omega) U.$$ 

(20)

In the above expression $\Phi(\omega)$ is a diagonal matrix with the $l$th diagonal element

$$\Phi_l(\omega) = 1 - \frac{1}{M_l(-\omega)\gamma_l[0]}$$

where $M_l(\omega)$ is a stable one-sided FT

$$M_l(\omega) = \sum_{n=0}^{\infty} \gamma_l[n]e^{-jn\omega}$$

(21)

and

(22)

which is uniquely determined by

$$M_l(\omega)M_l(-\omega) = \frac{d_l}{p_l}(\omega) + 1.$$ 

(23)

The direct current (dc) component $\gamma_l[0]$ in $M_l(\omega)$ can be found by

$$\gamma_l[0] = \exp\left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \ln \left[ \frac{d_l}{p_l}(\omega) + 1 \right] d\omega \right\}.$$ 

(24)

The $d_l$'s and $u_l$'s are the corresponding eigenvalues and eigenvectors of the frequency-domain correlation matrix $R_f$, defined as

$$R_f \triangleq \begin{pmatrix} r_f[0] & r_f[1] & \cdots & r_f[K-1] \\ r_f[-1] & r_f[0] & \cdots & r_f[K-2] \\ \vdots & \vdots & \ddots & \vdots \\ r_f[-K+1] & r_f[-K+2] & \cdots & r_f[0] \end{pmatrix}.$$ 

(25)

$U = (u_1, \ldots, u_K)$ is obviously a unitary matrix.

The MMSE channel estimator derived from (20) is shown in Fig. 2. The unitary linear inverse transform $U^H$ and transform $U$ in the figure perform the eigendecomposition of the frequency-domain correlation. The estimator turns off the zero or small $d_l$'s to reduce the estimation noise. For those large $d_l$'s, linear filters are used to take advantage of the time-domain correlation.

It should be noted that a channel estimator described in [5] uses only the frequency-domain correlation functions. However, the MMSE estimator described here exploits the channel correlations in both the time- and frequency-domains, resulting in better performance than the estimator in [5].

As shown in the appendix, the average MMSE of the channel estimator is

$$\text{MMSE} \triangleq \frac{1}{K} \sum_{k=1}^{K} E[\hat{H}[n,k] - H[n,k]]^2 = \frac{\rho}{K} \sum_{l=1}^{K} \left( 1 - \exp\left\{ -\frac{1}{2\pi} \int_{-\pi}^{\pi} \ln \left[ \frac{d_l}{p_l}(\omega) + 1 \right] d\omega \right\} \right).$$

(26)

For Jakes’ model, $p_l(\omega) = \rho_j(\omega)$. Then by direct calculation

$$\phi_l = 1 - \left( \frac{\alpha_l}{2} \right)^{\omega_l(\pi)} \exp\left\{ -\frac{\omega_l(\pi)}{\pi} \right\} \phi_l.$$ 

(27)

$$\text{MMSE}_J(\omega_l) \triangleq \frac{\rho_j}{K} \sum_{k=1}^{K} \left( 1 - \left( \frac{\alpha_l}{2} \right)^{\omega_l(\pi)} \exp\left\{ -\frac{\omega_l(\pi)}{\pi} \right\} \right)$$

(28)
where

$$\alpha_l \triangleq \frac{2d_l}{\omega_d \rho}$$  \hspace{1cm} (29)

$$\kappa(\alpha_l) \triangleq \left\{ \begin{array}{ll}
\frac{\pi}{2} \alpha_l + \sqrt{1 - \alpha_l^2} \ln \frac{1 + \sqrt{1 - \alpha_l^2}}{\alpha_l}, & \text{if } \alpha_l < 1 \\
\frac{\pi}{2} \alpha_l - \sqrt{1 - \alpha_l^2} - 1 \left( \frac{\pi}{2} - \arcsin \frac{1}{\alpha_l} \right), & \text{if } \alpha_l \geq 1.
\end{array} \right.$$  \hspace{1cm} (30)

If the time-domain correlation is ideal $\omega_d$-band-limited, i.e.,

$$p_B(\omega) \triangleq \left\{ \begin{array}{ll}
\frac{\pi}{\omega_d}, & \text{if } |\omega| \leq \omega_d \\
0, & \text{otherwise}
\end{array} \right.$$  \hspace{1cm} (31)

then

$$\phi_l = 1 - \frac{1}{\left(1 + \frac{\pi d_l}{\omega_d \rho} \frac{\omega_d}{\pi} \right)}$$  \hspace{1cm} (32)

$$\text{MMSE}_B(\omega_d) = \frac{\rho}{K} \sum_{l=1}^{K} \left( 1 - \frac{1}{\left(1 + \frac{\pi d_l}{\omega_d \rho} \frac{\omega_d}{\pi} \right)} \right).$$  \hspace{1cm} (33)

For any $\omega_d$-band-limited function $p_t(\omega)$ satisfying

$$\frac{1}{2\pi} \int_{-\omega_d}^{\omega_d} p_t(\omega) d\omega = 1$$  \hspace{1cm} (34)

we have

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \ln \left[ \frac{d_l}{\rho} \left( p_t(\omega) + 1 \right) \right] d\omega - \frac{1}{2\pi} \int_{-\pi}^{\pi} \ln \left[ \frac{d_l}{\rho} \left( p_B(\omega) + 1 \right) \right] d\omega = \frac{1}{2\pi} \int_{-\omega_d}^{\omega_d} \ln \left[ \frac{d_l}{\rho} \left( p_t(\omega) - \frac{\pi}{\omega_d} \right) + 1 \right] d\omega$$

$$\leq \frac{1}{2\pi} \int_{-\omega_d}^{\omega_d} \frac{d_l}{\rho} \left( p_t(\omega) - \frac{\pi}{\omega_d} \right) d\omega = 0$$  \hspace{1cm} (35)

with equality if and only if $p_t(\omega) = p_B(\omega)$ almost everywhere.

In the above derivation the inequality $\ln (x + 1) \leq x$ for all $x > -1$ has been used. Hence

$$\text{MMSE}(\omega_d) = \frac{\rho}{K} \sum_{l=1}^{K} 1 - \exp \left\{ -\frac{1}{2\pi} \int_{-\pi}^{\pi} \ln \left[ \frac{d_l}{\rho} \left( p_t(\omega) + 1 \right) \right] d\omega \right\}$$

$$\leq \frac{\rho}{K} \sum_{l=1}^{K} 1 - \exp \left\{ -\frac{1}{2\pi} \int_{-\pi}^{\pi} \ln \left[ \frac{d_l}{\rho} \left( p_B(\omega) + 1 \right) \right] d\omega \right\}$$

$$= \text{MMSE}_B(\omega_d).$$  \hspace{1cm} (36)

The above inequality suggests that a channel with an ideal band-limited time-domain correlation gives the worst mean-square-error (MSE) performance among all channels.

### IV. Robust Channel Estimator Design

Once the channel statistics, such as the time-domain correlation and frequency-domain correlation, are known, the optimum channel estimator can be designed. However, in mobile wireless links the channel statistics depend on the particular environments, for example, indoor or outdoor, urban or suburban, and change with time. Hence, it is not robust to design a channel estimator that tightly matches the channel statistics.

In this section we first analyze the performance degradation due to a mismatch of the estimator to the channel statistics and develop a robust estimator design approach. Then, we briefly discuss the design of finite-tap robust estimators.

#### A. Mismatch Analysis

If an MMSE channel estimator is designed to match a channel with time- and frequency-domain correlations $\tau_t[m]$ and $\Phi(\omega)$, respectively, then its coefficients $\bar{\tau}_t[m, l, k]$ are determined from (20) by

$$\bar{\mathbf{C}}(\omega) = \mathbf{U}^H \bar{\Phi}(\omega) \mathbf{U}$$  \hspace{1cm} (37)

where the definitions of $\mathbf{U}$ and $\bar{\Phi}(\omega)$ are similar to those of $\mathbf{U}$ and $\Phi(\omega)$ except that $\tau_f[l]$ and $r_f[l]$ there are replaced by $\bar{\tau}_{t_1}[m]$ and $\bar{\tau}_{f_1}[l]$, respectively.

For a channel with time- and frequency-domain correlations $\tau_t[m]$ and $r_f[l]$, rather than $\bar{\tau}_t[m]$ and $\bar{\tau}_f[l]$, from (16), the MSE for the designed channel estimator is

$$\text{MSE}((\bar{\tau}_t[m, l, k])) = \sum_{k=1}^{K} \sum_{m=0}^{M} \sum_{l=-\infty}^{\infty} \left( \bar{\tau}_t[m, l, k] - \delta[m, l] \right)^2 + \rho \sum_{m=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \left( \bar{\tau}_t[m, l, k] \right)^2$$

$$= \sum_{m, l=\infty}^{\infty} \sum_{k=1}^{K} \left( \bar{\tau}_t[m, l, k] - \delta[m, l] \right) \text{Tr} \left( \mathbf{C}[\tau_t[m]] \mathbf{C}^H[m] \right)$$

$$+ \rho \sum_{m=\infty}^{\infty} \text{Tr} \left( \mathbf{C}[\tau_t[m]] \mathbf{C}^H[m] \right)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} p_t(\omega) \text{Tr} \left( \bar{\mathbf{C}}(\omega) \mathbf{I} \mathbf{R}_f(\bar{\mathbf{C}}(\omega) - \mathbf{I})^H \right) d\omega$$

$$+ \rho \frac{1}{2\pi} \int_{-\pi}^{\pi} \text{Tr} \left( \bar{\mathbf{C}}(\omega) \mathbf{C}^H(\omega) \right) d\omega$$  \hspace{1cm} (38)

Substituting (37) into (38), we obtain a general formula for

$$\delta[n] = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{otherwise.} \end{cases}$$  \hspace{1cm} (39)
the MSE of the mismatched channel estimator

\[
\text{MSE}(\{\tilde{m}, l, k\})\]

\[
= \frac{1}{2\pi} \int_{-\pi}^{\pi} p_h(\omega) \text{Tr}\{U^H \Phi(\omega) U - I\} \cdot R_f(U^H \Phi(\omega) U - I) \text{d}\omega
\]

\[
+ \frac{\rho}{2\pi} \int_{-\pi}^{\pi} \text{Tr}\{U^H \Phi(\omega) U(U^H \Phi(\omega) U)^H\} \text{d}\omega
\]

\[
= \frac{1}{2\pi} \int_{-\pi}^{\pi} p_h(\omega) \text{Tr}\{\tilde{U} R_f U^H (\Phi(\omega) - I)\} \cdot (\Phi(\omega) - I) \text{d}\omega
\]

\[
+ \frac{\rho}{2\pi} \int_{-\pi}^{\pi} \text{Tr}\{\Phi(\omega)(\Phi(\omega))^H\} \text{d}\omega.
\]

(40)

1) Time-Domain Correlation Mismatch: Assume that the frequency-domain correlation of the channel estimator matches that of the channel, i.e.,

\[
\tau_f[k] = \tau_f[k]
\]

(41)

for \(k = 1, \ldots, K\), and the time-domain correlation \(\tau_f[n]\) is mismatched. Then

\[
\tilde{U} R_f \tilde{U}^H = D = \text{diag}\{d_1, \ldots, d_K\}
\]

(42)

and

\[
\text{MSE}(\{\tilde{m}, l, k\})
\]

\[
= \sum_{k=1}^{K} \frac{d_k}{2\pi} \int_{-\pi}^{\pi} p_h(\omega) |\tau_f(\omega) - 1|^2 \text{d}\omega
\]

\[
+ \sum_{k=1}^{K} \frac{\rho}{2\pi} \int_{-\pi}^{\pi} |\Phi_f(\omega)|^2 \text{d}\omega
\]

\[
= \sum_{k=1}^{K} \frac{d_k}{2\pi} \int_{-\pi}^{\pi} [p_h(\omega) - \bar{p}_h(\omega)] |\Phi_f(\omega) - 1|^2 \text{d}\omega
\]

\[
+ \sum_{k=1}^{K} \frac{\rho}{2\pi} \int_{-\pi}^{\pi} |\Phi_f(\omega)|^2 \text{d}\omega
\]

\[
= \sum_{k=1}^{K} \frac{d_k}{2\pi} \int_{-\pi}^{\pi} [p_h(\omega) - \bar{p}_h(\omega)] |\Phi_f(\omega) - 1|^2 \text{d}\omega
\]

\[
+ \text{MMSE}.
\]

(43)

The first term in the above expression represents the MSE variation due to the mismatch. By means of (24) and (21),\(\text{MSE}(\{\tilde{m}, l, k\})\) can be further simplified into

\[
\text{MSE}(\{\tilde{m}, l, k\})
\]

\[
= \sum_{k=1}^{K} \frac{\rho}{2\pi} \frac{1}{\gamma_k(0)} \int_{-\pi}^{\pi} \frac{[p_h(\omega) - \bar{p}_h(\omega)]}{d_\text{f}(\omega) + \rho} \text{d}\omega
\]

\[
+ \text{MMSE}.
\]

(44)

Hence, if an OFDM channel estimator is designed using \(p_B(\omega)\) as the time-domain correlation, then the time-domain correlation mismatch of the estimator will not degrade its performance. This suggests that a robust channel estimator should use \(p_B(\omega)\) as the time-domain correlation.

2) Frequency-Domain Correlation Mismatch: To analyze the frequency-domain correlation mismatch, we assume that the time-domain correlation of the designed estimator is the same as that of the channel, that is, \(\tilde{p}_f(\omega) = p_f(\omega)\), and the frequency-correlation matrix of the designed estimator has the same eigenvectors as that of the channel. That is, \(R_f\) can be eigendecomposed into

\[
R_f = U^H D U \quad \text{or} \quad \tilde{U} R_f \tilde{U}^H = D
\]

(48)

where \(D = \text{diag}\{d_1, \ldots, d_K\}\) and \(\Sigma_k d_k = K\). \(d_k\) and \(\tilde{d}_k\) for \(k = 1, \ldots, K\) are generally different.

Although the second assumption seems strange, it is, in fact, reasonable. As indicated in [5], with tolerable leakage, both matrices \(\tilde{U}\) and \(U\) can be approximated by the discrete FT (DFT) matrix that is defined as

\[
W \triangleq \frac{1}{\sqrt{K}} \begin{bmatrix}
1 \\
1 e^{2\pi i/K} \\
\vdots \\
1 e^{2\pi i(K-1)/K}
\end{bmatrix}
\]

(49)

The leakage of the above approximation depends on the guard interval and the channel multipath delay profile. If the delay \(\tau\) of a path is an integer multiple of the sampling interval \(t_s\), that is, \(\tau = lt_s\), then all of the energy from the path will be mapped to \(d_k\). Otherwise, if the delay is a noninteger multiple of the sampling interval, that is, \((l-1)t_s < \tau < lt_s\), then most of its energy will be contained in \(d_{l-1}\) and \(d_l\), although the energy will leak to all \(d_k\)’s. Hence, if the maximum delay spread is \(t_{\text{sp}}\) then for all \(l \leq K\), \((K_\text{s} = [Kt_{\text{sp}}/T_s])\), \(d_l \approx 0\), where \(T_s\) is the symbol duration of OFDM.
Applying the above two assumptions to (40), we get

\[
\text{MSE (} \{ \rho[m, l, k] \} \text{)} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left\{ \mathbf{D}_{\mathbf{t}}(\omega) - \mathbf{I} \right\} \mathbf{D}^H(\omega) \, d\omega + \frac{\rho}{2\pi} \int_{-\pi}^{\pi} \mathbf{D}_{\mathbf{t}}(\omega) \mathbf{D}^H(\omega) \, d\omega
\]

\[
= \sum_{l=1}^{K} \frac{1}{2\pi} \int_{-\pi}^{\pi} d\tilde{p}_l(\omega) \tilde{p}_l(\omega) - 1^2 \, d\omega + \sum_{l=1}^{K} \frac{\rho}{2\pi} \int_{-\pi}^{\pi} \left| \mathbf{D}_{\mathbf{t}}(\omega) \right|^2 \, d\omega
\]

\[
= \sum_{l=1}^{K} (d_l - \tilde{d}_l) \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{p}_l(\omega) \tilde{p}_l(\omega) - 1^2 \, d\omega + \text{MMSE}
\]

\[
\Gamma(d_l) = \frac{1}{2\pi^2 T_0^2} \int_{-\pi}^{\pi} \frac{p_k(\omega)}{d_l \tilde{p}_l(\omega) + 1} \, d\omega.
\]

In the above derivation we have used (23) and (21). From its definition, \( \Gamma(0) = 1 \) and \( \Gamma(x) < 1 \) for any \( x > 0 \).

If the channel estimator is designed such that

\[
\tilde{d}_l = \begin{cases} K/K_0, & \text{for } 1 \leq l \leq K_0 \\ 0, & \text{for } K_0 + 1 \leq l \leq K \end{cases}
\]

then, for any channel with \( d_l = 0 \) for \( K_0 + 1 \leq l \leq K \) and \( \sum_{l=1}^{K_0} d_l = K \), we have

\[
\text{MSE (} \{ \rho[m, l, k] \} \text{)} = \text{MMSE}.
\]

However, if

\[
\sum_{l=1}^{K_0} d_l < K
\]

then

\[
\text{MSE (} \{ \rho[m, l, k] \} \text{)} > \text{MMSE}
\]

since \( \Gamma(x) < \Gamma(0) \) for any \( x > 0 \).

From the above discussion, a robust estimator, which is insensitive to the channel statistics, should have an ideal band-limited time-domain correlation \( \rho_{[t]} = F^{-1} \{ \rho_{[\omega]} \} = (\sin (\omega d \delta f) / \omega d \delta f) \) and a frequency-domain correlation matrix

\[
\begin{bmatrix}
\rho_{[t][0]} & \cdots & \rho_{[t][K-1]} \\
\vdots & \ddots & \vdots \\
\rho_{[t][-K+1]} & \cdots & \rho_{[t][0]}
\end{bmatrix} = \mathbf{W}^H \mathbf{D} \mathbf{W}
\]

where \( \mathbf{D} = \text{diag} \{ K/K_0, \cdots, K/K_0, 0, \cdots, 0 \} \), \( K_0 = \lceil \Delta f t_{\text{max}} K \rceil = \lceil t_{\text{max}} / T_s K \rceil \), and \( \omega d = 2 \pi f_{\text{max}} \). Note that \( \mathbf{W} \) is the DFT matrix defined in (49). In this case the average MSE of the robust estimator is

\[
\text{MMSE}_B = \frac{K_0 \rho}{K} \left( 1 - \frac{1}{\left| \frac{\pi K_0}{K_0 \omega d \delta f} + 1 \right| \omega d / \pi} \right).
\]

For any channel with \( f_d \leq f_{\text{max}} \) and \( t_d \leq t_{\text{max}} \), the average MSE should be \( \text{MMSE}_B \).

Fig. 3 shows the MSE of the robust estimator that matches different Doppler frequencies and delay spreads. As shown, the MSE is almost a constant if \( t_{\text{max}} f_{\text{max}} \) is fixed. In particular, let

\[
\mu \triangleq \frac{\omega_d K_0}{\pi} \rho \approx \frac{2 \pi f_{\text{max}} f_{\text{max}}}{\eta f_{\text{s}} T_s}
\]

where \( \eta f_{\text{s}} = 1 / T_s \) is the baud of the OFDM system. For OFDM systems satisfying \( \mu \ll 1 \) and \( (\omega_d / \pi) \ln (1 / \mu) \gg 1 \), the average MSE of the robust channel estimator is

\[
\text{MMSE}_B = \frac{K_0 \rho}{K} \left( 1 - \frac{1}{\left| \frac{\pi K_0}{K_0 \omega d \delta f} + 1 \right| \omega d / \pi} \right)
\]

\[
= \frac{K_0 \rho}{K} \left( 1 - \exp \left\{ -\frac{\omega_d}{\pi} \ln \left( \frac{\pi K_0}{K_0 \omega d \delta f} + 1 \right) \right\} \right) \approx \frac{K_0 \rho \omega d}{K} \ln \left( \frac{\pi K_0}{K_0 \omega d \delta f} + 1 \right) \approx \mu \ln \left( \frac{1}{\mu} \right) \approx 1.
\]

If the channel estimator is designed to match the Doppler spectrum given by (10) and the \( d_l \)'s given by (52), then

\[
\alpha_l = \begin{cases} \frac{2K}{\omega_d K_0 \rho}, & \text{for } 1 \leq l \leq K_0 \\ \alpha, & \text{for } K_0 + 1 \leq l \leq K \end{cases}
\]

and therefore, from (28), we have

\[
\text{MMSE}_J = \frac{K_0 \rho}{K} \left( 1 - \frac{\alpha}{2} \right)^{-(\omega_d / \pi)} \exp \left\{ -\frac{\omega_d}{\pi} b(\alpha) \right\}
\]

\[
= \frac{K_0 \rho}{K} \left( 1 - \exp \left\{ -\frac{\omega_d}{\pi} \ln \left( \frac{\alpha}{2} + b(\alpha) \right) \right\} \right) \approx \mu \ln \frac{1}{\mu} + 1 \approx \mu \ln \left( \frac{1}{\mu} - 0.1447 \right).
\]

Hence, compared with the estimator that tightly matches the Doppler spectrum, the performance degradation of the robust channel estimator is negligible.

B. FIR Channel Estimator

Here, we briefly introduce the design of finite-tap robust estimators. Note that from the previous section, for a robust
Fig. 3. Normalized MSE of channel estimator versus Doppler frequency (a) when SNR = 10 dB and \( f_d = 0, 5, 10, 20, \) and 40 Hz and (b) when SNR = 10 dB and \( f_{D,D} = 0.00008, 0.0008, \) and 0.008, respectively.

Let

\[
\bar{R}_f = W^H \bar{D}W \\
\tau^*_f[n] = \frac{\sin(n\omega_d)}{n\omega_d}.
\]

with length \( L \), we have

\[
\sum_{m=-(L-1)}^{0} \tilde{\tau}_{f}[n-m,k]^{T} \tau_f[n+1,k] + \rho \tilde{\tau}_{f}[n,k] = 0
\]

for \( n = 0, -1, \ldots, -(L-1) \), where

\[
\tilde{\tau}_{f}^n = \begin{cases} 
K_o/K, & \text{if } l \leq K_o \\
0, & \text{otherwise}.
\end{cases}
\]

From (66), it is obvious that \( \tilde{\tau}_{f}[n,k] = 0 \) for \( l > K_o \). For \( l \leq K_o \)

\[
\tilde{\tau}_{f}[n,k] = \frac{\tilde{\tau}_f[n,k]}{K_o/K} \tau_n
\]

where

\[
(c_0, c_1, \ldots, c_{L-1})^T = \left( R_t + \frac{K_o^2 I}{K} \right)^{-1} \bar{\tau}_f
\]

\[
R_t = \begin{bmatrix}
\bar{\tau}_f[0] & \bar{\tau}_f[1] & \cdots & \bar{\tau}_f[L-1] \\
\bar{\tau}_f[-1] & \bar{\tau}_f[0] & \cdots & \bar{\tau}_f[L-2] \\
\vdots & \vdots & \ddots & \vdots \\
\bar{\tau}_f[-L+1] & \bar{\tau}_f[-L+2] & \cdots & \bar{\tau}_f[0]
\end{bmatrix}
\]

(70)

Using derivations similar to those in Section III, the FT of the coefficient matrix of the designed FIR channel estimator is determined by

\[
\bar{\mathcal{C}}(\omega) = W^H \bar{\Phi}(\omega)W
\]

where \( \bar{\Phi}(\omega) \) is a diagonal matrix with

\[
\bar{\Phi}(\omega) = \text{diag} \{ c(\omega), \ldots, c(\omega), 0, \ldots, 0 \}
\]

and \( c(\omega) \) is the FT of \( c_n \). The estimation error of the FIR estimator can be found by

\[
\text{MSE} = \frac{K_o c(\omega)}{K}.
\]

Hence, for the robust FIR channel estimator, the \( U \) in Fig. 2 is the DFT matrix \( W \) and the \( \Phi_k(\omega) \)'s for \( k = 1, \ldots, K \) are \( c(\omega) \).

In Fig. 4 the average MSE of a robust FIR channel estimator is shown as a function of its length. From the figure, for the estimator matching a 40-Hz maximum Doppler frequency, a 50-tap FIR estimator is needed to exploit the time-domain correlation of the channel parameters, while for one matching a 200-Hz maximum Doppler frequency, only a five-tap channel estimator is sufficient.

V. REFERENCE GENERATION AND COMPUTER SIMULATION

In this section we demonstrate the performance improvement of an OFDM system with our robust channel estimator. First, we briefly describe the simulated OFDM system.
A. System Parameters

In our simulation, we use a two-path Rayleigh-fading channel model [12] with delay from 0 to 40 μs and Doppler frequency from 10 to 200 Hz. The channels corresponding to different receivers have the same statistics. Two antennas are used for receiver diversity.

To construct an OFDM signal, assume that the entire channel bandwidth, 800 kHz, is divided into 128 subchannels. The four subchannels on each end are used as guard tones and the rest (120 tones) are used to transmit data. To make the tones orthogonal to each other, the symbol duration is 160 μs. An additional 40-μs guard interval is used to provide protection from ISI due to channel multipath delay spread. This results in a total block length \( T_f = 200 \mu s \) and a subchannel symbol rate \( r_b = 5 \text{ kBD} \).

To compare the performance of the OFDM system with and without the channel estimation, PSK modulation with coherent demodulation and differential PSK (DPSK) modulation with differential demodulation are used, respectively. As in [4], a (40,20) RS code, with each code symbol consisting of three quadrature PSK/differential quadrature PSK (QPSK/DQPSK) symbols grouped in frequency, is used in the system. Hence, each OFDM block forms an RS codeword. The RS decoder erases ten symbols, based on signal strength, and corrects five additional random errors. Hence, the simulated system can transmit data at 1.2 Mb/s before decoding or 600 kb/s after decoding, over an 800-kHz channel.

B. Reference Generation

An ideal reference is assumed in the derivation of the channel estimator in Section III. In practical systems a reference can be generated during a training block. In subsequent blocks a reference is generated using the received signals. We consider four possible reference generating schemes.

1) Undecoded/Decoded Dual Mode Reference: If the RS decoder can successfully correct all errors in an OFDM block, the reference for the block can be generated by the decoded data; hence \( \hat{a}[n,k] = a[n,k] \). Otherwise, \( \hat{a}[n,k] = \hat{a}[n,k] \).
2) Undecoded Reference: \( \hat{a}[n,k] = \hat{a}[n,k] \), no matter whether the RS decoder can successfully correct all errors in a block or not.
3) Decoded/CMA Dual Mode Reference: The constant modulus algorithm (CMA) is one of the most effective adaptive blind equalization algorithms [13], [14]. It can be also used to generate a reference for the
Fig. 5. (a) WER and (b) NMSE of robust estimator with different references versus SNR for the system without training blocks when the 50-tap channel estimator matches the channel with $f_d = 40$ Hz and $t_d = 20$ μs.

OFDM channel estimator. If the RS decoder can successfully correct all errors in a block, the reference for the block can be generated from the decoded data; hence $\hat{a}[n,k] = d[n,k]$. Otherwise, the reference can use the projection of $y[n,k]$ on the unit circle, i.e., $\hat{a}[n,k] = y[n,k]/|y[n,k]|$.

4) Error Removal Reference: If the RS decoder can successfully correct all errors in a block, the reference for the block can be generated by the decoded data. Otherwise, the $H[n-1,k]$’s are used instead of $\hat{H}[n,k]$’s for $k = 1, \cdots, K$, respectively.

C. Simulation Results

Figs. 5–8 demonstrate the performance of the channel estimator using different references under different channel conditions. To get insight into the average behavior of the channel estimator, we have averaged the performance over 10 000 OFDM blocks.

Fig. 6. (a) WER and (b) NMSE of robust estimator with different references versus SNR for the system with 1% training blocks when the 50-tap channel estimator matches the channel with $f_d = 40$ Hz and $t_d = 20$ μs.

Fig. 5 shows the word-error rate (WER) and normalized MSE (NMSE) for the OFDM system with only the first block as a training or synchronizing block and for a channel with $f_d = 40$ Hz and $t_d = 20$ μs. From the figure, when SNR ≥ 8 dB, the channel estimator using the decoded/undecoded dual mode reference or the undecoded reference can estimate the channel parameters with NMSE as small as −22 dB. Hence, the WER of the system is almost the same as the performance assuming ideal knowledge of the channel parameters, which is about 3 dB better than using differential detection. However, due to error propagation through the references, there is a noise threshold for the channel estimator. For the estimators with the decoded/undecoded dual mode reference and the undecoded reference, the noise thresholds are 7 and 8 dB, respectively. Once the SNR is larger than the noise threshold, the system performance will be significantly better than that of the OFDM system using DQPSK.
Fig. 7. (a) WER and (b) NMSE of robust estimator with different references versus SNR for the system with 1% training blocks when the 50-tap channel estimator matches the channel with $f_d = 200$ Hz and $t_d = 5$ μs.

To suppress the error propagation, training blocks are periodically inserted in the data stream. Fig. 6 illustrates the WER and NMSE for the systems with 1% training blocks. In this situation the noise threshold disappears and, hence, the OFDM system with channel estimation has better performance than the one without channel estimation when the SNR ranges from 0 to 20 dB.

For a channel with a Doppler frequency as large as 200 Hz, as indicated in Fig. 7, if 1% training blocks are used, then the OFDM system with channel estimation has about a 1.5-dB SNR improvement compared to the one without channel estimation. If the training blocks are increased from 1% to 10% of the data, then the required SNR for WER = $10^{-1}$ is reduced by 1 dB compared with the one with 1% training blocks, but the required SNR for WER = $10^{-2}$ is almost the same as before, as shown in Fig. 8.

Figs. 9 and 10 illustrate the robustness of our channel estimator. As indicated in the analysis in Section IV, if a channel estimator is designed to match the channel with 40-Hz maximum Doppler frequency and 20-μs maximum delay spread, then for all channels with $f_d \leq 40$ Hz and $t_d \leq 20$ μs, the system performance is not worse than the channel with $f_d = 40$ Hz and $t_d = 20$ μs, as indicated by Fig. 9. However, for channels with $f_d > 40$ Hz or $t_d > 20$ μs, such as $f_d = 80$ Hz and $t_d = 20$ μs or $f_d = 40$ Hz and $t_d = 40$ μs, the system performance degrades dramatically. On the other hand, as indicated in Fig. 10, if the estimator is designed to match a Doppler frequency or delay spread larger than the actual ones, the system performance degrades only slightly compared with estimation that exactly matches the channel Doppler frequency and delay spread.

VI. CONCLUSIONS

We have presented the design of robust channel estimators for OFDM systems that make full use of the time- and frequency-correlations of the rapid dispersive fading wireless channel and are insensitive to the channel statistics. Computer simulation demonstrates that channel estimation gives about 2.5-dB improvement when the Doppler frequency is
40 Hz, and about 1.5-dB improvement when the Doppler frequency is as large as 200 Hz. This channel estimation used together with antenna arrays has the potential to provide significant suppression of cochannel interference in OFDM systems.

**APPENDIX**

**DERIVATION OF MMSE CHANNEL ESTIMATOR**

By the orthogonality principle [15], the $c[m, l, k]$'s are determined by

$$E\{\hat{H}[n, k]H[n, k]\} = 0.$$  \hspace{1cm} (A.1)

Direct calculation shows that (A.1) is equivalent to

$$E\left\{ \left( \sum_{m_1=-\infty}^{0} \sum_{l_1=(K-k)}^{k-1} c[m_1, l_1, k]H[n-m_1, l] \right) \hat{H}[n-m, k-l] \right\} + \rho c[m, l, k] = 0$$

or

$$\sum_{m_1=-\infty}^{0} \sum_{l_1=(K-k)}^{k-1} c[m_1, l_1, k]r_{H}[m-m_1, l-l_1] = -r_{H}[m, l] + \rho c[m, l, k] = 0 \hspace{1cm} (A.2)$$

for $m = \cdots, -1, 0$ and $l = -(K-k), \cdots, 0, \cdots, (k-1)$, where

$$\rho \triangleq E[w[n, k]a^*[n, k]]^{2} = E[w[n, k]]^{2}. \hspace{1cm} (A.3)$$

With the separation property (7) of the correlation of the channel frequency response, (A.2) can be simplified to

$$\sum_{m_1=-\infty}^{0} \sum_{l_1=(K-k)}^{k-1} r_l[m-m_1]r_j[l-l_1]c[m_1, l_1, k] = -r_j[m]r_l[l] + \rho c[m, l, k] = 0 \hspace{1cm} (A.4)$$
Let
\[ \mathbf{r}_f[k] \triangleq \begin{pmatrix} r_f[k-1] \\ \vdots \\ r_f[0] \\ r_f[-K+k] \end{pmatrix} \]
\[ \mathbf{c}[m,k] \triangleq \begin{pmatrix} c[m,k-1,k] \\ \vdots \\ c[m,0,k] \\ c[m,-K+k,k] \end{pmatrix} \]
\[ R_f = (\mathbf{r}_f[1], \mathbf{r}_f[2], \ldots, \mathbf{r}_f[K]). \]

Then, (A.4) can be written in vector form as
\[ \sum_{m=-\infty}^{0} r_t[m-m_1] \mathbf{R}_f \mathbf{c}[m,k] - r_t[m] \mathbf{r}_f[k] + \mathbf{c}[m,k] = 0. \]  
\[ \text{(A.7)} \]

Let the eigendecomposition of \( R_f \) be
\[ R_f = \mathbf{U}^H \mathbf{D} \mathbf{U} \]  
\[ \text{(A.8)} \]
where \( \mathbf{U} \) is a unitary matrix and \( \mathbf{D} \) is a diagonal matrix with diagonal elements \( d_t \). It is clear that
\[ \sum_{i=1}^{K} d_t = K r_f[0] = K. \]  
\[ \text{(A.9)} \]

Substituting (A.8) into (A.7) gives
\[ \sum_{m=-\infty}^{0} r_t[m-m_1] \mathbf{U}^H \mathbf{D} \mathbf{U} \mathbf{c}[m,k] - r_t[m] \mathbf{r}_f[k] + \mathbf{c}[m,k] = 0. \]  
\[ \text{(A.10)} \]

Consequently
\[ \sum_{m=-\infty}^{0} \{ r_t[m-m_1] \mathbf{U} \mathbf{c}[m,k] \} - r_t[m] \mathbf{D}^l \mathbf{U} \mathbf{r}_f[k] + \mathbf{D}^l \mathbf{U} \mathbf{c}[m,k] = 0. \]  
\[ \text{(A.11)} \]

The \( \mathbf{D}^l \) in the above expression is the pseudoinverse of \( \mathbf{D} \), which is also a diagonal matrix with elements
\[ d_t^l = \begin{cases} 1/d_t & \text{if } d_t \neq 0 \\ 0 & \text{if } d_t = 0. \end{cases} \]  
\[ \text{(A.12)} \]

Denote
\[ \mathbf{c}[m,k] = \mathbf{U} \mathbf{c}[m,k] \quad \mathbf{r}_f[k] = \mathbf{U} \mathbf{r}_f[k] \]
\[ \text{(A.13)} \]
then
\[ \sum_{m=-\infty}^{0} r_t[m-m_1] \mathbf{c}[m,k] - r_t[m] \mathbf{D}^l \mathbf{r}_f[k] + \rho \mathbf{D}^l \mathbf{c}[m,k] = 0. \]  
\[ \text{(A.14)} \]

or
\[ \sum_{m=-\infty}^{0} r_t[m-m_1] \mathbf{c}[m,l,k] - r_t[m] \mathbf{d}_l \mathbf{r}_f[l,k] + \rho \mathbf{d}_l \mathbf{c}[m,l,k] = 0. \]  
\[ \text{(A.15)} \]

for \( m = \ldots, -1, 0 \), where \( \mathbf{d}_l \mathbf{r}_f[l,k] \) and \( \mathbf{c}[m,l,k] \) are the \( l \)th elements of \( \mathbf{r}_f[k] \) and \( \mathbf{c}[m,k] \), respectively.

For the \( l \) with \( d_l \neq 0 \), from [16, Appendix A]
\[ \mathbf{c}(\omega;l,k) \triangleq \sum_{n=-\infty}^{0} \mathbf{c}[n,l,k] e^{-j \omega n} \]  
\[ = \mathbf{r}_f[l,k] l^l \left( 1 - \frac{1}{M_l(-\omega)^\gamma[0]} \right) \]  
\[ \text{(A.16)} \]
where \( M_l(\omega) \) is a stable one-sided FT
\[ M_l(\omega) = \sum_{n=0}^{\infty} \gamma[n] e^{-j \omega n} \]  
\[ \text{(A.17)} \]
which is uniquely determined by
\[ M_l(\omega) M_l(-\omega) = \frac{d_l}{d_t} \rho n(\omega) + 1. \]  
\[ \text{(A.18)} \]
The dc component \( \gamma[0] \) in \( M_l(\omega) \) can be found by
\[ \gamma[0] = \exp \left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \ln \left[ \frac{d_l}{d_t} \rho n(\omega) + 1 \right] d\omega \right\}. \]  
\[ \text{(A.19)} \]
For the \( l \) with \( d_l = 0 \), we have \( \mathbf{c}[m,l,k] = 0 \) for \( m = \ldots, -1, 0 \), which can be also written as (A.16).

Therefore, from (A.13)
\[ \mathbf{c}(\omega;k) \triangleq \sum_{n=-\infty}^{0} \mathbf{c}[n,k] e^{-j \omega n} \]  
\[ = \mathbf{U}^H \mathbf{c}(\omega;k) \]  
\[ = \mathbf{U}^H \hat{\mathbf{r}}(\omega) \mathbf{D}^l \mathbf{r}_f[k] \]  
\[ = \mathbf{U}^H \hat{\mathbf{r}}(\omega) \mathbf{D}^l \mathbf{U} \mathbf{r}_f[k] \]  
\[ \text{(A.20)} \]
where \( \hat{\mathbf{r}}(\omega) \) is a diagonal matrix with the \( l \)th diagonal element
\[ \hat{\mathbf{r}}(\omega)_l = \frac{1}{M(\omega)} \]  
\[ \text{(A.21)} \]
which is zero for the \( l \) with \( d_l = 0 \). If we define
\[ \mathbf{C}(\omega) \triangleq (\mathbf{c}(\omega;1), \mathbf{c}(\omega;2), \ldots, \mathbf{c}(\omega;K)) \]  
\[ \text{(A.22)} \]
The MMSE channel estimator for OFDM systems derived (A.23) is shown in Fig. 2.

Substituting (A.2) into (16), the MMSE for the estimation of $H[n,k]$ is

$$\text{MMSE}_k = \mathbb{E}[\hat{H}[n,k]-H[n,k]]^2 = [0,0,k]_L. \tag{A.24}$$

Let

$$\text{MMSE} = \begin{pmatrix} \text{MMSE}_1 \\ \text{MMSE}_2 \\ \vdots \\ \text{MMSE}_K \end{pmatrix}. \tag{A.25}$$

Then, from (A.23)

$$\text{MMSE} = \frac{\rho}{2\pi} \int_{-\pi}^{\pi} \text{diag} \{C(\omega)\} d\omega$$

$$= \frac{\rho}{2\pi} \int_{-\pi}^{\pi} \text{diag} \{U^H \Phi(\omega)U\} d\omega$$

$$= \rho \text{diag} \{U^H \Phi U\} \tag{A.26}$$

where $\Phi$ is a diagonal matrix with the $l$th diagonal element

$$\phi_l = \frac{1}{2\pi} \int_{-\pi}^{\pi} 1 - \frac{1}{M_l(\omega)\gamma_l[0]} d\omega$$

$$= 1 - \frac{1}{\gamma_l[0]}$$

$$= 1 - \exp \left\{ -\frac{1}{2\pi} \int_{-\pi}^{\pi} \ln \left[ \frac{d}{\rho} p_l(\omega) + 1 \right] d\omega \right\}. \tag{A.27}$$

The average MMSE is

$$\text{MMSE} = \frac{1}{K} \sum_{l=1}^{K} \text{MMSE}_l$$

$$= \frac{\rho}{K} \text{Tr} \{U^H \Phi U\} = \frac{\rho}{K} \sum_{l=1}^{K} \phi_l$$

$$= \frac{\rho}{K} \sum_{l=1}^{K} \left( 1 - \exp \left\{ -\frac{1}{2\pi} \int_{-\pi}^{\pi} \ln \left[ \frac{d}{\rho} p_l(\omega) + 1 \right] d\omega \right\} \right). \tag{A.28}$$

where $\text{Tr}\{\cdot\}$ denotes the trace of matrix, defined as the sum of the diagonal elements of the matrix.

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