1.15: A car with a dead battery is connected to a car with a good battery, as shown below, to start the car with the dead battery. Positive terminals are connected together; negative terminals are connected together.

The current $i$ flows from car B to car A; $i$ is measured and found to be 30 A.

a) Which car has the dead battery?
- $i$ flows into the + terminal of car A.
- Thus the power absorbed by car A is $p = VI$ using the passive sign convention.

$p = vi = 12(30) = 360$ W
- $p$ is positive (360 W), so car A is absorbing energy. Car B is supplying energy. Car A is the car with the dead battery.

b) If this connection is maintained for 1 minute, how much energy is transferred to the battery?
- $p = vi = dw/dt$;
- $dw = pdt$ or $\Delta W = p\Delta t$
- $\Delta W = $ energy transferred to battery in 1 minute

$\Delta t = 1$ minute = 60 sec

$\Delta W = p\Delta t = vi \Delta t = 12(30)60 = 21,600$ J = 21.6 kJ

Or you can integrate:

$dW = pdt \rightarrow \int_{t=0}^{t=60} dw = \int_{t=0}^{t=60} pdt = \int_{t=0}^{t=60} 360 dt = 360(60) = 21,600$ J

$W(t=1\text{ min}) - W(t=0\text{ min}) = \Delta W$ in 1 minute = 21,600 J = 21.6 kJ
HW 1, EE 188, cont.: Note: do either 1.17 or 1.22, not both

1.17: \( v(t) = 0 \), \( t < 0 \); \( i(t) = 0 \), \( t > 0 \)
\[ v(t) = e^{-500t} - e^{-1500t} V, \quad t \geq 0 \]
\[ i(t) = 30 - 40e^{-500t} + 10e^{-1500t} \text{ mA}, \quad t \geq 0 \]

a) Find power at \( t = 1 \) ms.
- Assume \( i(t) \) flowing into + terminal
- \( p(t) = v(t) \cdot i(t) \)
- \( p(t = 1 \text{ ms}) = v(t = 1 \text{ ms}) \cdot i(t = 1 \text{ ms}) \)
- \( p(\infty) = (e^{-500(\infty)} - e^{-1500(\infty)}) (30 - 40e^{-500(\infty)} + 10e^{-1500(\infty)}) \)
- \( p(t = \infty) = 0.0031 \text{ W} = 3.1 \text{ mW} \)

b) How much energy is delivered to circuit element between \( 0 \) and \( 1 \) ms?
\[ \Delta W = \int_{t=0}^{1 \text{ ms}} p(t) \, dt \]
\[ p(t) = \int_{t=0}^{1 \text{ ms}} \left( e^{-500t} - e^{-1500t} \right) (30 - 40e^{-500t} + 10e^{-1500t}) \, dt \]
\[ \Delta W = \int_{0}^{1 \text{ ms}} \left( 30e^{-500t} - 40e^{-1000t} - 30e^{-1500t} + 50e^{-2000t} - 10e^{-3000t} \right) \, dt \]
\[ \Delta W = \left. \left[ \frac{30e^{-500t}}{-500} - \frac{40e^{-1000t}}{-1000} - \frac{30e^{-1500t}}{-1500} + \frac{50e^{-2000t}}{-2000} - \frac{10e^{-3000t}}{-3000} \right] \right|_{0}^{1 \text{ ms}} \]
\[ \Delta W = 0.001235 \text{ J} \approx 0.001245 \text{ J} \]

Energy delivered btw. 0 and 1 ms: \( \Delta W = 0.00124 \text{ J} = 1.24 \text{ mJ} \)
1.7. c. Find the total energy delivered to the circuit.

Total energy \( W = \int_0^\infty P(t) \, dt = \int_0^\infty V(t)I(t) \, dt \)

\[
W = \int_0^\infty \left( 30e^{-50t} - 40e^{-100t} + 30e^{-150t} + 50e^{-200t} - 10e^{-300t} \right) \, dt
\]

\[
= \left[ \frac{30e^{-50t}}{-50} - \frac{40e^{-100t}}{-100} - \frac{30e^{-150t}}{-150} + \frac{50e^{-200t}}{-200} - \frac{10e^{-300t}}{-300} \right]_0^\infty
\]

\( e^{-\infty} = 0 \quad e^{-0t} = 1 \)

\[
W = \frac{30}{500} - \frac{40}{1000} - \frac{30}{1500} + \frac{50}{2000} - \frac{10}{3000}
\]

\[
W = 180 - 120 - 60 + 75 - 10 = \frac{65}{3000} \text{ mJ} = 0.0217 \text{ mJ}
\]

\[
W = 0.0217 \text{ mJ} = 2.17 \times 10^{-5} \text{ J}
\]
1.29: Check the interconnection below and state whether the total power delivered (supplied) equals the total power absorbed. Current and voltage values for each element are given in the table below.

![Diagram of electrical circuit]

<table>
<thead>
<tr>
<th>Element</th>
<th>Voltage (V)</th>
<th>Current (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1.6</td>
<td>80</td>
</tr>
<tr>
<td>b</td>
<td>2.6</td>
<td>60</td>
</tr>
<tr>
<td>c</td>
<td>-4.2</td>
<td>-50</td>
</tr>
<tr>
<td>d</td>
<td>1.2</td>
<td>20</td>
</tr>
<tr>
<td>e</td>
<td>1.8</td>
<td>30</td>
</tr>
<tr>
<td>f</td>
<td>-1.8</td>
<td>-40</td>
</tr>
<tr>
<td>g</td>
<td>-3.6</td>
<td>-30</td>
</tr>
<tr>
<td>h</td>
<td>3.2</td>
<td>-20</td>
</tr>
<tr>
<td>i</td>
<td>-2.4</td>
<td>30</td>
</tr>
</tbody>
</table>

*Passive Sign Convention:
* i flowing into + terminal, $p = V_i$
* i flowing into - terminal, $p = -V_i$

Calculate power used by each element:
* if power $p$ is positive, is power absorbed
* if power $p$ is negative, is power delivered (supplied)
\[ P_a = -V_{ia} = -1.6(-08) = -0.128 \text{ W} \]
\[ P_b = -V_{ib} = -2.6(-06) = -0.156 \text{ W} \]
\[ P_c = V_{ic} = (4.2)(-05) = 0.21 \text{ W} \]
\[ P_d = -V_{id} = (1.2)(-08) = -0.024 \text{ W} \]
\[ P_e = V_{ie} = 1.8(-03) = 0.054 \text{ W} \]
\[ P_f = V_{if} = (-1.8)(-04) = -0.072 \text{ W} \]
\[ P_g = V_{ig} = (-3.6)(-03) = -0.108 \text{ W} \]
\[ P_h = V_{ih} = (3.2)(-02) = -0.064 \text{ W} \]
\[ P_j = V_{ij} = (-2.4)(-03) = -0.072 \text{ W} \]

Power delivered = \[P_a + P_b + P_d + P_f + P_h\]
(all negative power) = -0.444 \text{ W}

Power absorbed = \[P_c + P_e + P_g + P_j\]
(all positive power) = +0.444 \text{ W}

Power delivered = power absorbed in magnitude but is opposite in sign, as it should be. Thus, we have conservation of power.
\[ P_{\text{Total}} = P_{\text{del}} + P_{\text{abs}} = 0 \]
The voltage and current at the terminals of an automobile battery during a charge cycle are shown below.

\[ V \text{ (in V)} \]

\[ I \text{ (in A)} \]

\[ t \text{ (ks)} \]

\[ i = dq/dt, \quad dq = idt, \quad Q = \int_i(t) \, dt = \int_{t=0}^{15 \text{ ks}} i(t) \, dt \]

**Non-integral Method**

Sum up area under \( i(t) \) curve

**Step 1**

\[ \text{Area} \Delta y = \frac{1}{2} \Delta y \, \Delta x = \frac{1}{2} (15-10)(4-0) \text{ ks} \]

\[ = \frac{1}{2} (5 \cdot 4) = 10 \text{ ks} \]

\[ = 10,000 \text{ C} \]
b) Calculate the total energy transferred to the battery.

\[ P = \frac{W}{t} = \frac{dW}{dt}, \quad dW = Pdt = v(t)i(t)dt \]

\[ \int_{t=0}^{t=15kS} dW = \int_{t=0}^{t=15kS} W(15kS) - W(0) = \Delta W = \text{total energy transferred to battery, because} \]

\[ i(t) = 0 \text{ for } t > 15kS \text{ so} \]

\[ P(t) = 0 \text{ for } t > 15kS \]

The easiest way to do this is to find equations for \( v(t) \) and \( i(t) \), as they are linear functions.

\[ v(t) = \begin{cases} 0 & \text{for } t < 0 \\ 9V & \text{for } 0 \leq t \leq 15kS \\ 12V & \text{for } t > 15kS \end{cases} \]
22(b) cont. \( V(t) = 0.002t + 9V \) satisfies the conditions that \( V(0) = 9V \) and \( V(15,000) = 12V \)

\[
i(t) = \begin{cases} 
15 - 0.00125t, & 4 \text{ks} \leq t \leq 0 \text{ks} \\
12 - 5 \times 10^{-4}t, & 12 \text{ks} \leq t \leq 24 \text{ks} \\
30 - 0.002t, & 15 \text{ks} \leq t \leq 12 \text{ks} \\
0, & t \geq 15 \text{ks}
\end{cases}
\]

where the equations for \( i(t) \) satisfy the endpoint values in the region they are used.

\[
p(t) = V(t) \cdot i(t) = \begin{cases} 
(0.0002t + 9)(15 - 0.00125t) = 13.5 - 0.00825t - 3.5 \times 10^{-7}t^2, & \text{for } 4 \text{ks} \leq t \leq 0 \text{ks} \\
(0.0002t + 9)(12 - 5 \times 10^{-4}t) = 108 - 0.0081t - 1 \times 10^{-7}t^2, & \text{for } 12 \text{ks} \leq t \leq 24 \text{ks} \\
(0.0002t + 9)(30 - 0.002t) = 270 - 0.012t - 4 \times 10^{-7}t^2, & \text{for } 15 \text{ks} \leq t \leq 12 \text{ks} \\
0, & \text{for } t \geq 15 \text{ks}
\end{cases}
\]

- Can either integrate \( p(t) \) to get \( \Delta W \), or approximate \( p(t) \) by dropping the \( t^2 \) term, as a piecewise linear function and add up area underneath \( p(t) \).

1. **Integration method:**

\[
\int_{t_1}^{t_2} A \, dt = A(t_2 - t_1)
\]

\[
\int_{t_1}^{t_2} A \, dt = \frac{At^2}{2} \bigg|_{t=t_1}^{t_2} = \frac{A}{2} (t_2^2 - t_1^2)
\]

\[
\int_{t_1}^{t_2} At^2 \, dt = \frac{At^3}{3} \bigg|_{t=t_1}^{t_2} = \frac{A}{3} (t_2^3 - t_1^3)
\]
1.22 (b), cont. Method 1: Integration

\[ \Delta W = \int_0^{4\text{ks}} p(t) \, dt + \int_4^{12\text{ks}} p(t) \, dt + \int_{12}^{15\text{ks}} p(t) \, dt \]

\[ = \int_0^{4\text{ks}} (135 - 0.00825 t - 2.5 \times 10^{-7} t^2) \, dt \]

\[ + \int_{12}^{15\text{ks}} (108 - 0.0021 t - 1 \times 10^{-7} t^2) \, dt \]

\[ + \int_{12}^{15\text{ks}} (270 - 0.012 t - 4 \times 10^{-7} t^2) \, dt \]

\[ = 135 \frac{(4000)}{2} - 0.00825 \frac{(4000)^2}{2} - 2.5 \times 10^{-7} \frac{(4000)^3}{3} \]

\[ + 108 \frac{(8000)}{2} - 0.0021 \frac{(12000)^2}{2} - 1 \times 10^{-7} \frac{(12000)^3}{3} \]

\[ + 270 \frac{(3000)}{2} - 0.012 \frac{(15000)^2}{2} - 4 \times 10^{-7} \frac{(15000)^3}{3} \]

\[ = 5400000 - 660000 - 53333 + 8640000 - 134400 - 55487 + 810000 - 4860000 - 219600 = 1,247,200 \text{ J} \]

\[ \Delta W = 1,247,200 \text{ J} = 1.247 \text{ MJ} \]

Method 2: Approximate \( p(t) \) as piecewise linear function by eliminating the \( t^2 \) terms. \( p'(t) \)

1.22 (b), cont. We need \( p(t) \) to match up at the boundaries. So let's calculate what \( p(t=0), p(t=4\text{ks}), p(t=12\text{ks}) \) are, fit a line between each of those.

\[ p(t=0) = V(t=0) i(t=0) = 9.15 = 9.15 \text{ W} \]

\[ p(t=4\text{ks}) = V(t=4\text{ks}) i(t=4\text{ks}) = (9.8)(10) = 98 \text{ W} \]

\[ p(t=12\text{ks}) = V(t=12\text{ks}) i(t=12\text{ks}) = (11.4)(6) = 68.4 \text{ W} \]

\[ p(t=15\text{ks}) = V(t=15\text{ks}) i(t=15\text{ks}) = (12)(0) = 0 \text{ W} \]
1.22(b) cont.

Plot $p(t)$ vs $t$: 135W of $p(t)$, in W

Note: this is a linear approximation to $p(t)$, not $p(t)$ itself.
EE 1988, HW #1, cont:

1.22(b), cont. Now sum up the area under the p(t) curve.

1. **Area 1:**

\[
\text{Area} = \frac{1}{2} \Delta P \Delta t = \frac{1}{2} (135 - 98)(4000 - 0) = \frac{37(4000)}{2} = 74000 \text{ J}
\]

2. **Area 2:**

\[
\text{Area} = \Delta P \Delta t = 98(4000) = 392000 \text{ J}
\]

3. **Area 3:**

\[
\text{Area} = \frac{1}{2} \Delta P \Delta t = \frac{1}{2} (98 - 68.4)(4000) = \frac{1}{2} (29.6)(8000) = 118400 \text{ J}
\]

4. **Area 4:**

\[
\text{Area} = \Delta P \Delta t = 68.4(8000) = 547200 \text{ J}
\]

5. **Area 5:**

\[
\text{Area} = \frac{1}{2} \Delta P \Delta t = \frac{1}{2} (68.4)(3000) = 102600 \text{ J}
\]

\[
\Delta W = \text{Total area} = 74000 + 392000 + 118400 + 547200 + 102600
\]

\[
\Delta W = 8841400 \text{ J} = 8.84 \times 10^6 \text{ J}
\]

\[
\Delta W = 1234200 \times 10^3 \text{ J} = 1.234 \times 10^6 \text{ J}
\]

\[
\Delta W' \neq \Delta W \text{ but is close}
\]

\[
\text{this is an approx. to } \Delta W, \text{ because } p' \text{ is approx to } p.
\]