Problems 4.47, 4.60 + 4.68:

4.47: Use mesh-current method to find total power dissipated in circuit below.

![Circuit Diagram]

2 meshes:

\[ a + b \]

+ 2 mesh currents

\[ i_a + i_b \]

Note that the current source is the only circuit elements between the 2 middle nodes. Thus we can remove it and create a single supermesh.

![Modified Circuit Diagram]

We still include the 2 mesh currents \( i_a + i_b \) from the earlier meshes.

\[ \text{KVL around supermesh:} \]

\[ -18 + 3i_a + 9i_b - 15 + 6i_b + 2i_a = 0 \]

\[ 5i_a + 15i_b = 33 \text{ V} \quad \text{Eqn} 1 \]

We also have a constraint on \( i_a + i_b \) due to the current source:

\[ \text{KCL: } -i_a - 3A + i_b = 0 \]

\[ \text{Eqn 2} \]

\[ i_b - i_a = 3A \]

or \[ i_b = 3 + i_a \quad \text{Eqn 2} \]

\[ 5i_a + 45 + 15i_a = 33 \]

\[ 20i_a = -12 \text{ V}; i_a = \frac{-12}{20} = -0.6 \text{ A} \]

\[ i_b = 3 + 0.6 = 3.6 \text{ A} \]

\[ i_a = -0.6 \text{ A}, i_b = 3.4 \text{ A} \]
4.47 (cont.): To find total power dissipated (absorbed) in circuit, we need to calculate the power dissipated (absorbed) by every resistor (and by any voltage source where positive current is going into the positive voltage terminal).

\[ P_{\text{bias}} = i^2R = (-0.6)^2 \cdot 3 = 1.08 \, \text{W} \]

\[ P_{R_1} = (i_a)^2 \cdot 3 = (-0.6)^2 \cdot 3 = 1.08 \, \text{W} \]

\[ P_{R_2} = (i_b)^2 \cdot 2 = (-0.6)^2 \cdot 2 = 0.72 \, \text{W} \]

\[ P_{R_9} = (i_c)^2 \cdot 9 = (2.4)^2 \cdot 9 = 51.84 \, \text{W} \]

\[ P_{R_6} = (i_d)^2 \cdot 6 = (2.4)^2 \cdot 6 = 34.56 \, \text{W} \]

Notice that, while I've drawn \( i_a \) in the clockwise direction, so that it enters the - terminal of the 18V source, \( i_a = -0.6 \, \text{A} \) is negative. In reality, a positive current of +0.6A flows into the + terminal of the 18V source. This means that the voltage source is not supplying power but is dissipating or absorbing power.

The power dissipated by the 18V source is

\[ P_{18V} = V_i - V_i = (18V)(-0.6A) = -10.8 \, \text{W} \quad \text{for } i_a \text{ into } - \text{ terminal} \]

\[ P_{18V} = V_i = V(-i_a) = 18(0.6A) = +10.8 \, \text{W} \quad \text{for } -i_a \text{ into } + \text{ terminal} \]

The 15V source is supplying power, because \( i_b \) flows into the - terminal of the 15V source. Similarly, the current source is flowing from ground + thus is in the direction of voltage rise (towards + potential) and is also supplying, not dissipating, power.
The total power dissipated:

\[ P_{\text{diss}} = P_{R_2} + P_{R_4} + P_{R_6} + P_{R_8} + P_{R_{10}} = 99 \, \text{W} \] (absorbed)

4.60 a) Find the current in the 10 kΩ resistor in the circuit below by making a succession of appropriate source transformations.

Note: Voltage source has terminal at top; current direction is from - to + so current direction is downward.

\[ I_S = \frac{100 \, \text{V}}{20 \, \text{kΩ}} = 5 \, \text{mA} \]

\[ 20 \, \text{kΩ} \parallel 80 \, \text{kΩ} = \frac{1600 \, \text{kΩ}^2}{100 \, \text{kΩ}} = 16 \, \text{kΩ} \]

\[ V_S = (5 \, \text{mA}) \times 16 \, \text{kΩ} = 80 \, \text{V} \]
4.60a (cont.)

\[ 16kΩ + 3kΩ + 1kΩ \]
\[ \text{in series} = 20kΩ \]
\[ \frac{80V}{20kΩ} \]
\[ I_s = 4mA \]

Now transform to current source

\[ 80V \]
\[ 20kΩ \]
\[ I_s = \frac{V_s}{R_s} = \frac{80V}{20kΩ} \]
\[ I_s = 4mA \]

2 current sources + 2 resistors in \( \parallel \):

Current sources add:
\[ I_{total} = (-4 + 12) mA = 8mA \]
\[ R_{eq} = \frac{80kΩ \parallel 60kΩ}{80kΩ + 60kΩ} = \frac{1200kΩ}{80kΩ + 60kΩ} = 15kΩ \]

\[ 8mA \]
\[ 15kΩ \]
\[ 10kΩ \]

Note that 5kΩ + 10kΩ are in series → same current \( i_0 \) flows through both of them → combine into single \( R_{eq} = 5kΩ + 10kΩ = 15kΩ \)

Use current divider:
\[ i_0 = I_s \frac{(15kΩ)}{(15kΩ + 15kΩ)} = I_s \]
\[ i_0 = 4mA \]

b. Using result from (a), work back through circuit to find power developed by 100V source.
To find power developed by 100 V source, we need to find current flowing through 100 V source (finding equivalent resistance is a problem due to current source).

Let's redraw the original circuit, labeling voltages and currents.

By KVL, \(-V_2 + i_0 (15 \text{k}\Omega) = 0\) so \(V_2 = 4 \text{mA} (15 \text{k}\Omega)\)

\(V_2 = 60 \text{ V}\)

Ohm's Law: \(V_2 = i_2 (60 \text{k}\Omega) = 60 \text{ V}; \ i_2 = \frac{60 \text{ V}}{60 \text{k}\Omega} = 1 \text{ mA}\)

By KCL at node 2: \(i_3 = 12 \text{ mA} + i_2 + i_0 = 0\)

\(i_3 = 12 \text{ mA} - i_2 - i_0 = (12 - 1 - 4) \text{ mA} = 7 \text{ mA}\)

Note that KCL at node 0 gives something: \(i_4 = i_3 = 7 \text{ mA}\)

KVL: \(-V_1 - i_3 (3 \text{k}\Omega) + V_2 = -i_0 (1 \text{k}\Omega) = 0\)

\(V_1 = V_2 - i_3 (4 \text{k}\Omega) = 60 - (7 \times 10^{-3})(4000) = 60 - 28 = 32 \text{ V}\)

\(V_1 = 32 \text{ V}\)

KVL: \(+100 \text{ V} - (20 \text{k}\Omega) i_s + V_1 = 0; i_s = \frac{132 \text{ V}}{20 \text{k}\Omega} = 6.6 \text{ mA}\)

or by KCL: \(i_s + i_4 - i_3 = 0; i_s = i_3 - i_4 = i_3 - V_1 / 80 \text{k}\Omega\)

\(i_s = 7 \text{ mA} - \frac{32}{80} = (7 - 0.4) \text{ mA} = 6.6 \text{ mA}\)

\(i_s = 6.6 \text{ mA}\)

\(P = -V i_s = (-100)(6.6 \text{ mA}) = -660 \text{ mW} = -0.66 \text{ W} = P_{100 \text{ V}}\)

or \(P = +660 \text{ mW developed/supplied by 100 V source}\)
4.68(a): Find the Thévenin equivalent w.r.t. terminals a,b for the circuit below by finding the open-circuit voltage \( V_{oc} \) and the short-circuit current \( I_{sc} \).

1. Find open-circuit voltage \( V_{oc} = V_{ab} = V_{th} \).
   - **Ohm's Law**: \( V_{oc} = 60i_a \) \( \text{Eqn } 1 \)
   - **KVL around outside loop**: 
     \[-9 + 20i_2 + V_{oc} + 10i_a = 0 \]
     \[20i_2 = 9 - V_{oc} - 10i_a = 9 - 60i_a + 10i_a = 9 - 70i_a \]
     \[i_2 = (9 - 70i_a)/20 \]

2. Apply KCL at node a:
   \[-1.8 - i_2 + i_a = 0 ; \]
   \[i_a = i_2 + 1.8 \] \( \text{Eqn } 3 \)

Substitute \( \text{Eqn } 2 \) into \( \text{Eqn } 3 \):

\[ i_a = (9 - 70i_a)/20 + 1.8 \quad \text{or} \quad 20i_a = 9 - 70i_a + 36 \]
\[ 90i_a = 45 \quad \text{or} \quad i_a = 1/2 \text{A} \]

From \( \text{Eqn } 1 \), \( V_{oc} = 60i_a = 60/2 \text{A} = 30\text{V} = V_{th} \)

3. Find short-circuit current \( I_{sc} \).
Note that there will be no current through the 60 Ω resistor; the 60 Ω resistor is in \( \parallel \) with \( R_{ab} = R_{sc} = 0 \) and thus \( i_{60Ω} = \frac{\text{in}(R_{60Ω})}{60 + R_{60Ω}} = 0 \) A. (current divider)

(All the current flows through the short circuit).

\[ i_{60Ω} = \frac{V_{ab}}{60} = 0 \]

- **Apply KCL at node a:**
  \[ -1.8 = i_2 + i_{60Ω} + i_{sc} = 0 \]
  or \( i_{sc} = 1.8 + i_2 + i_{60Ω} \)

  \[ i_{sc} = 1.8 + i_2 \]

- **Apply KCL at node b:**
  \[ i_{sc} = i_{60Ω} + i_{10Ω} = 0 \]
  \[ i_{60Ω} = i_{sc} + i_{10Ω} = i_{sc} \]

  \[ i_{10Ω} = i_{sc} \]

- **Apply KVL around outside loop:**
  \[ -9 + 20i_2 + V_{ab} = 0 \]
  \[ V_{ab} = 0 \] because short circuit

  \[ 20i_2 = 9 - V_{ab} \]
  \[ -10i_{sc} = 9 - 10i_{sc} \]

  \[ i_2 = \frac{9 - 10i_{sc}}{20} \]
- Substitute eqn 3 into eqn 1
\[ i_{sc} = 1.8 + 1.5 = 1.8 + 9 - 10i_{sc} = 1.8 + 0.45 - 0.5i_{sc} \]
\[ 1.5i_{sc} = 2.25 \]
\[ i_{sc} = \frac{2.25}{1.5} = 1.5 \text{A} \]

\[ R_{TH} = \frac{V_{TH}}{i_{sc}} = \frac{30 \text{V}}{1.5 \text{A}} = 20 \Omega = R_{TH} \]

**Thévenin Equivalent Circuit:**

\[ V_{TH} = 30 \text{V}, \quad R_{TH} = 20 \Omega \]

b) Solve for the Thévenin resistance \( R_{TH} \) by removing the independent sources. Compare your result to the Thévenin resistance found in (a).

- Remove the independent sources by shorting out the voltage source \( V \) and converting the current source to an open circuit.

- To make things easier, label all the nodes and redraw the circuit.
The 5Ω + 25Ω resistors have no effect on the circuit. Let's examine the node C area closer:

From this view, it's clear that we have
\[
\frac{5Ω + 25Ω}{R_{sc}} = \frac{30Ω}{10} = 3Ω
\]

We redrew the circuit as

\[
R_{th} = R_{eq} = 20Ω
\]

Same result for \(R_{th}\) as found in part (a). Finding \(R_{eq}\) was easier than finding \(I_{sc}\).