9.3: At $t = -250/6 \mu s$, a sinusoidal voltage is known to be zero and going positive. The voltage is next zero at $t = 1250/6 \mu s$. It is also known that the voltage is 75V at $t = 0$.

a) What is the frequency of $v$ in hertz?

$$f = \frac{1}{T}; T = \text{period} = \text{time in which} \ v \ \text{repeats itself}.$$ 

$$v(t = -250/6 \times 10^{-6} s) = 0 = v(t = +1250/6 \times 10^{-6} s)$$

$v$ repeats in time 

$$T = \frac{1250}{6} \times 10^{-6} \text{ s} - (-250/6 \times 10^{-6} \text{ s}) = \frac{1250}{6} \times 10^{-6} \text{ s} = 250 \times 10^{-6} \text{ s} = 250 \mu \text{ s} = 0.25 \text{ ms}.$$ 

$$f = \frac{1}{T} = \frac{1}{0.25 \times 10^{-3} \text{ s}} = 4 \text{000 cycles/s} = 4 \text{000 Hz}.$$ 

b) What is the expression for $v$?

$v(t)$ is sinusoidal, so $v(t)$ has the form of 

$$v(t) = V_m \cos (\omega t + \phi)$$

$$\omega = 2\pi f = 4000 \pi \text{ rad/s}$$

so 

$$v(t) = V_m \cos (4000 \pi t + \phi)$$

$$\phi = \frac{\pi}{2}; v(t = -250/6 \times 10^{-6} \text{ s}) = 0$$

$+$ going positive 

$\cos \Theta = 0$ at $\Theta = \pm \pi/2$ (and multiples thereof)

$v(t)$ is going positive at $t = -250/6 \times 10^{-6} \text{ s}$

Comparing plot of $v(t)$ to plot of $\cos(\Theta)$, we see that $v(t = -250/6 \times 10^{-6} \text{ s})$ corresponds to $V_m \cos(-\pi/2)$.

$4000 \pi t + \phi = -\pi/2$

$\phi = -\pi/2 - 4000 \pi \times (-250/6 \times 10^{-6} \text{ s})$

$= -\pi/2 + \pi/6 = \frac{-3\pi}{6} + \pi/6 = -\frac{2\pi}{6} = -\pi/3.$
9.3 (cont.) \( v(t) = V_m \cos(4000\pi t - \pi/3) \)

\( V_m = \frac{V}{2} \) \( v(t=0) = 75 \text{ V} \) \( V_m \neq 75 \) because \( \cos \) is shifted by \(-\pi/3\), so \( v(t) \) is not max at \( t=0 \). However, we can find out from the equation what \( v(t) \) equals at \( t=0 \), and solve for \( V_m \).

\[ v(t=0) = V_m \cos(4000\pi \cdot 0 - \pi/3) = V_m \cos(-\pi/3) \]

\( \cos(-\pi/3) = \cos(-60^\circ) = \frac{1}{2} \)

so \( V(t=0) = \frac{V_m}{2} = 75 \text{ V} \); \( V_m = 2(75) \text{ V} = 150 \text{ V} \)

\[ v(t) = 150 \cos(4000\pi t - \pi/3) \]

or if we want to write \( v(t) \) as a sine instead of cosine, using \( \sin(wt) = \cos(wt - \pi/2) \)

\[ v(t) = 150 \sin(4000\pi t - \pi/3 + \pi/2) \]

\[ = 150 \sin(4000\pi t - 2\pi/6 + 3\pi/6) \]

\[ v(t) = 150 \sin(4000\pi t + \pi/6) \]

9.7: The rms value of the sinusoidal voltage supplied to the convenience outlet of a U.S. home is 120 V. What is the maximum value of the voltage at the outlet?

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ \text{or} \]

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]

\[ V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \]

\[ V_{\text{max}} = \sqrt{2} V_{\text{rms}} = \sqrt{2} \cdot 120 \text{ V} \]
9.22: a) For the circuit shown below, find the frequency (in rad/s) at which the impedance \( Z \) is purely resistive.

\[
\begin{align*}
Z_{ab} &= Z_L + Z_R \frac{1}{Z_C} \\
Z_p &= Z_R \frac{1}{Z_C} \\
Z_p &= Z_R + Z_C \\
Z_L &= j\omega L = j5\omega \\
Z_R &= 4000 \\
Z_C &= \frac{1}{j\omega C} \\
Z_p &= \frac{4000}{j\omega C} \\
Z_p &= \frac{4000}{j1 + j4000\omega C} \\
\text{multiply top + bottom by } 1 - j4000\omega C \\
Z_p &= \frac{4000(1 - j4000\omega C)}{(1 + j4000\omega C)(1 - j4000\omega C)} \\
&= \frac{4000 - j16 \times 10^6 \omega C}{1 + 16 \times 10^6 \omega^2 C^2} \\
&= \frac{4000}{1 + 6.25 \times 10^6 \omega^2 C^2} - \frac{j10}{1 + 6.25 \times 10^6 \omega^2 C^2}
\end{align*}
\]
9.22. a (cont.):
\[ Z_{ab} = Z_L + Z_p = \frac{j 10 \omega}{1 + 6.25 \times 10^{-6} \omega^2} \]
\[ Z_{ab} = j 5 \omega + \frac{4000}{1 + 6.25 \times 10^{-6} \omega^2} \]
\[ = \frac{4000}{1 + 6.25 \times 10^{-6} \omega^2} + j \left( 5 \omega - \frac{10 \omega}{1 + 6.25 \times 10^{-6} \omega^2} \right) \]
resistive (real) part \hspace{1cm} \text{reactive (imag.) part}

For \( Z_{ab} \) to be purely resistive, the reactive part of \( Z_{ab} \) (Im \( Z_{ab} \)) must be 0.
\[ 5 \omega = \frac{10 \omega}{1 + 6.25 \times 10^{-6} \omega^2} \]
\[ 5 \omega (1 + 6.25 \times 10^{-6} \omega^2) = 10 \omega, \quad \text{or} \quad 1 + 6.25 \times 10^{-6} \omega^2 = 1 \]
\[ 3.125 \times 10^{-6} \omega^2 = \frac{1}{2}, \quad \text{or} \quad \omega^2 = 160,000 \, \text{rad}^2/\text{s}^2 \]
\[ \omega \approx 400 \, \text{rad/s} \quad \text{resonant freq.: freq. at which } Z_{ab} \text{ is purely resistive} \]

b) Find the value of \( Z_{ab} \) at the frequency of (a).
\[ Z_{ab} = \frac{4000}{1 + 6.25 \times 10^{-6} (400)^2} + j \left( 2000 - \frac{4000}{1 + 6.25 \times 10^{-6} (400)^2} \right) \]
\[ = \frac{4000}{2} + j (2000 - 4000/2) = 2000 + j 0 \]
\[ Z_{ab} = 2000 \, \Omega \]

\[ Z_{ab} = 2 \times 10^3 \, \Omega \]
9.23: Find the impedance $Z_{ab}$ in the circuit below. Express $Z_{ab}$ in both polar and rectangular form.

Note: Two impedances are in \( \parallel \) if they are connected to the same top node and same bottom node. Thus, the 20 $\Omega$ resistor and $j20$ inductor impedances are in \( \parallel \). Similarly, the series combination of $5-j30 \Omega$ is in \( \parallel \) with the series combo of $10+j10$. Let's reduce $Z_{ab}$ to the following:

where $Z_{ab} = 10 - j40 + Z_1 + Z_2$

$Z_1 = \frac{(5-j30)(10+j10)}{5-j30+10+j10} = \frac{50+300+j(50-300)}{15-j20} = \frac{350-j250}{15-j20} = 430.12e^{j35.54^\circ} \approx 17.2e^{j17.59^\circ} = 16.4 + j5.2$
9.23 (cont.) \( Z_2 = 10 + j10 \)

\[
Z_{ab} = 10 - j40 + Z_1 + Z_2 = 10 - j40 + 16.4 + j5.2 + 10 + j10
\]

\[
Z_{ab} = 10 + 16.4 + 10 + j(5.2 + 10 - 40) = 36.4 - j24.8
\]

\[
Z_{ab} = 36.4 - j24.8 = r \cdot e^{j\phi}, r = \sqrt{(36.4)^2 + (-24.8)^2} = 44.05
\]

\[
\phi = \tan^{-1}\left(\frac{-24.8}{36.4}\right) = -34.27^\circ
\]

9.28: The circuit below is operating in the sinusoidal steady state. Find the steady-state expression for \( V_o(t) \) if \( V_g = 64\cos 8000t \) V.

Sinusoidal steady state means that all voltages and currents in the circuit have had time to stabilize into their sinusoidal forms. We can represent them in phasor form and represent all passive elements as impedances.

The circuit diagram is shown, and the impedances are calculated as follows:

\[ V_g = 64\cos (8000t) = V_m\cos (\omega t) \] so \( \omega = 8000 \) rad/s.

Use impedances:

\[ Z_c = -j4000 \quad \Omega \]

\[ Z_L = jwL = j(8000)(5) \quad \Omega = j40000 \quad \Omega \]

Redraw circuit with impedances:

Phasor \( V_g = 64\angle 0^\circ \)
Vo is the voltage across the inductor and is also the voltage across the resistor. If we reduce the parallel combination of Zp and ZL to a single equivalent impedance Zp, then Vo will be the voltage across that impedance Zp.

\[ Z_p = Z_p/Z_L = Z_p/Z_L \frac{Z_p + Z_L}{Z_p + Z_L} = \frac{20000 + j4000}{20000 + j4000} \]

\[ Z_p = \frac{8 \times 10^6 e^{i(90^\circ - 63.4^\circ)}}{4472.1} = 1788.9 e^{i63.4^\circ} \]

\[ Z_p = 1.79 e^{i63.4^\circ} \] kΩ = 1.6 + j1.8 kΩ

\[ Z_{TOTAL} = Z_c + Z_p = -j4000 + 1600 + j800 = 1600 - j3200 \]

\[ Z_{TOTAL} = 3577.71 e^{i(-63.4^\circ)} \]

\[ I_g = \frac{V_g}{Z_{TOTAL}} = \frac{6440}{3577.71 e^{i(-63.4^\circ)}} = 1.79 e^{i63.4^\circ} \]

\[ I_g = 17.9 e^{i63.4^\circ} \] mA

\[ V_o = I_g Z_p = (17.9 e^{i63.4^\circ} \cdot 1.79 e^{i(126.57^\circ)}) = 32 \angle 90^\circ \] kV

\[ V_o = 32 \angle 90^\circ \] V = 32e^{j90^\circ} V

\[ V_o(t) = \text{Re}(V_o e^{j\omega t}) = \text{Re}(32e^{j90^\circ} e^{j\omega t}) = 32e^{j90^\circ} \]

\[ V_o(t) = 32 \cos(\omega t + 90^\circ), \] \( \omega = 8000 \) rad/s

\[ V_o(t) = 32 \cos(8000 \omega t + 90^\circ) \]