EE 188 Practice Problems for Exam 1, Spring 2009

1. Circle T (true) or F (false) for each of these Boolean equations.
   (a). T F Negative power indicates power delivered
   (b). T F The unit of charge is the amp
   (c). T F 0.005 V = 5 mV
   (d). T F Ohm's Law states that $v = iR$
   (e). T F Current is the rate of flow of charge

2. Equivalent Resistance: Find the equivalent resistance $R_{eq}$ of the resistive circuit below.

   \[
   R_{eq} = \frac{50 + 200}{2} || \left( \frac{100 + 200}{2} || \left( \frac{100 + 100}{2} \right) \right) + 100 \Omega
   \]

   \[
   \frac{200}{2} || 200 \Omega = \frac{400}{400} = 100 \Omega \quad \text{note that } \frac{R_1}{R_1} = \frac{R_1}{2}
   \]

   \[
   R_{eq} = \frac{50 + 200}{2} || (100 + 100) + 100 = \frac{50 + 200}{2} || 200 + 100 \Omega
   \]

   as above, \( \frac{200}{2} || 200 = 100 \Omega \)

   \[
   R_{eq} = \frac{50 + 100 + 100}{2} = 250 \Omega
   \]

   \[ R_{eq} = 250 \Omega \]
3. Ohm's Law and Power: Given the simple resistive circuit below:

3(a). Find the current $i$ flowing through the 400 kΩ resistor.

By KVL: $-25 \, V + 100 \times 10^3 i + 400 \times 10^3 i = 0$
$500 \times 10^3 i = 25$
$i = \frac{25}{500 \times 10^3} = \frac{1}{20} \times 10^{-3} \, A = 0.05 \, mA = 50 \, \mu A$

Is this the same current that flows through the 100 kΩ resistor?
(circle one): Yes [ ] No [ ]

3(b). Find the power absorbed or supplied by the 100 kΩ resistor. Negative power indicates power absorbed and positive power indicates power supplied.

* $i$ goes into + terminal of $V_{100k\Omega}$, so use $P = + i \, V$

$P = + i \, V_{100k\Omega} = + i^2 R = (5 \times 10^{-5})^2 (1 \times 10^5) = 25 \times 10^{-10} (1 \times 10^5)$
$P = 25 \times 10^{-5} \, W = 250 \, mW = 0.25 \, mW$

Power is (circle one): absorbed [ ] supplied [ ]
4. **Voltage Division:** Use voltage division to find the voltage $V_2$ in the circuit below.

\[ V_2 = \frac{10 \cdot 100}{50 + 100 + 100} = \frac{1000}{250} = 4 \text{ V} \]

Reduce circuit to series circuit:

\[ R_{eq} = \frac{500}{50} \parallel \frac{25}{25} \parallel \frac{50}{50} = \frac{500 \cdot 125}{500 + 125} = \frac{62500}{625} = 100 \Omega \]

By voltage division,

\[ V_2 = \frac{10 \cdot 100}{50 + 100 + 100} = \frac{1000}{250} = 4 \text{ V} \]

\[ V_2 = 4 \text{ V} \]
5. Current Division: Use current division to find the current $i$ flowing through the 25 Ω resistor.

For current division, reduce $R$ in branch with 25 Ω resistor to a single $R_{eq}$.

$I_s = 5 \text{ mA}$

\[ R_{eq} = \frac{50 + 25 + 50}{2} = 50 \text{Ω} \]

By current division, $i = \frac{I_s (500)}{500 + R_{eq}} \Rightarrow \frac{5 \text{ mA}(500)}{500 + 50} = \frac{2500}{550} = \frac{250}{55} = \frac{50}{11} = \frac{4}{11} (5 \text{ mA}) = \frac{4}{11} \text{ mA}$
6. **KVL, KCL and Dependent Current Source**: Use Kirchhoff’s Voltage Law (KVL) and Kirchhoff’s Current Law (KCL) to find the current flowing through the 25 Ω resistor, \( i_2 \).

![Circuit Diagram]

By **KCL at node b**, \(-2i_2 + i_1 + i_2 = 0\) so \( i_1 = i_2 \)

By **KVL from e \( \rightarrow \) d \( \rightarrow \) b \( \rightarrow \) c \( \rightarrow \) e**, 

\[-V_{de} - 10 + V_{bc} + V_{ce} = 0\]

\[-V_{de} = -i_1 \cdot 50 \quad V_{bc} = 75i_2 \quad V_{ce} = 25i_2\]

\[-50i_1 - 10 + 75i_2 + 25i_2 = -50i_1 - 10 + 100i_2 = 0\]

From KCL, \( i_1 = i_2 \) so 

\[-50i_2 - 10 + 100i_2 = 0\] or 

\[50i_2 = 10\]

\[i_2 = \frac{10}{50} = \frac{1}{5} A = 0.2 A = 200 \text{ mA} = i_2\]

\[i_2 = 0.2 \text{ A} = 200 \text{ mA}\]
7. Nodal Analysis (Node-Voltage Method): Find equations for the voltage $V_2$ and the current $i_1$ in the circuit below, using nodal analysis (node-voltage method):

7(a). First, label your nodes. Which nodes are essential nodes? Which node is your reference node? Indicate this below and on the circuit diagram.

Essential nodes: b and d

Reference node: d

7(b). Use KVL to find an equation for $V_2$ that includes the unknown current $i_1$.

- By Ohm's Law, $V_2 = 50 \cdot i_1$ (or by KVL: $d \rightarrow c \rightarrow b \rightarrow d$, $-V_2 + i_1 \cdot 50 = 0$)
- Also by KVL at nodes $d \rightarrow c \rightarrow b \rightarrow d$, $-25i_2 - 5i_1 + V_2 = 0$
- Either eqn is fine
- Also by KVL at nodes $d \rightarrow a \rightarrow b \rightarrow d$, $-10 + 20i + V_2 = 0$

$V_2 = 50i_1$ Volts or $V_2 = 5i_1 + 25i_2$ Volts

7(c). Now use KCL at the appropriate essential node to find an equation for $i_1$ in terms of $V_2$.

KCL at node b: $-i + i_2 + i_1 = 0$ $i_1 = i - i_2$

From 7(b), $i = \frac{10 - V_2}{20} + i_2 = \frac{V_2 - 5i_1}{25}$

Plug these into above eqn

$i_1 = \frac{10 - V_2}{20} - \frac{V_2}{25} + \frac{5i_1}{25}$

$i_1 \left(1 - \frac{5}{25}\right) = \frac{10}{20} - V_2 \left(\frac{1}{20} + \frac{1}{25}\right)$

$i_1 \left(\frac{4}{5}\right) = \frac{1}{2} - 0.9V_2$

$i_1 = \frac{5}{8} - 0.9V_2$ Amps

7(b) $20i = 10 - V_2$ in 7(c), you use $20i = 10 - 50i$ (since $V_2 = 50i$), your $i_1$ eqn will be different: $i_1 = \frac{5}{33} - 2V_2$ Amps

How? $i_1$ and $V_2$ are dependent. Both eqns result in the same $(i_1, V_2)$ solution.
7(d). **Extra:** Given your two equations above for $V_2$ and $i_1$, solve for both unknowns.

$$V_2 = 50 \cdot i_1 \quad \Rightarrow \quad \dot{i}_1 = \frac{5}{6} - \frac{9}{8} \cdot \frac{V_2}{80}$$

Plug eqn for $V_2$ into eqn for $\dot{i}_1$:

$$\dot{i}_1 = \frac{5}{6} - \frac{9}{8} \cdot \frac{(50 \cdot i_1)}{80} = \frac{5}{8} - \frac{450}{8} \cdot \frac{i_1}{8}$$

$$\dot{i}_1 \left(1 + \frac{45}{8}\right) = \frac{5}{8} \quad \Rightarrow \quad \frac{53}{8} \cdot \dot{i}_1 = \frac{5}{8} \quad \Rightarrow \quad \dot{i}_1 = \frac{5}{53} A$$

$$V_2 = 50 \cdot i_1 = \frac{50 \cdot (5)}{53} V = \frac{250}{53} V$$

$V_2 = \frac{250}{53} = 4.717$ Volts

$$i_1 = \frac{5}{53} = 0.09434$ Amps