EE 188 Practice Problems for Exam 3, Spring 2009

Include units in your answers where appropriate. Assume that all circuits are in sinusoidal steady state.

1. Circle T (true) or F (false) for each of these statements:
   (a). T F At the resonant frequency \( \omega_0 \), circuit impedance is purely real.
   (b). T F The RMS voltage value is greater than the maximum voltage value.
   (c). T F Capacitors in series add.
   (d). T F The average power of a purely capacitive load is zero.
   (e). T F An inductive load has a lagging power factor.

2. Equivalent Capacitance and Inductance:

2(a). Find the equivalent capacitance \( C_{eq} \) for the capacitive circuit below.

\[
\begin{array}{c}
15 \, \mu F \quad K \quad 5 \, \mu F \quad 5 \, \mu F \\
\end{array}
\]

- \( \text{caps in } || \text{ add} \)
- \( \text{caps in series:} \)

\[
\frac{1}{C_{eq}} = \text{sum of inverses, or} \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \text{ for } C_1 + C_2 \text{ in series}
\]

\[
C_{eq} = \frac{15 \, \mu F \text{ in series with } 5 \, \mu F || 5 \, \mu F}{5 \, \mu F || 5 \, \mu F = (5 + 5) \, \mu F = 10 \, \mu F}
\]

\[
15 \, \mu F \text{ in series with } 10 \, \mu F = \frac{(15 \times 10^{-6})(10 \times 10^{-6})}{(15 + 10) \times 10^{-6}} = \frac{150 \times 10^{-6}}{25} = \frac{6 \times 10^{-6}}{25} = 0.24 \, \mu F
\]

\[
C_{eq} = \frac{150}{25} \, \mu F = 6 \, \mu F
\]

\[
C_{eq} = 6 \, \mu F
\]
2(b). Find the equivalent inductance $L_{eq}$ for the inductive circuit below.

\[ L_{eq} = 15 \text{ mH} \]

3. Sinusoidal Voltage, Phasor Voltage and RMS Voltage:
A sinusoidal voltage is given by $v(t) = 25 \cos(2000t + \pi/3)$.

3(a). What is the phasor voltage $V$?

\[ V = V_m e^{j\phi} = 25 e^{j\pi/3} = 25 \angle \pi/3 \text{ rads} \]

\[ V = 25 \angle \pi/3 \text{ rad} \]

3(b). What is the radial frequency $\omega$?

\[ \omega = 2000 \text{ rad/s} \]

3(c). What is the phase offset $\phi$ in both radians and degrees?

\[ \text{angle in degrees} = \frac{180}{\pi} \times \left(\frac{180}{4}\right) = 60^\circ \]

\[ \phi = \pi/3 \text{ in radians} \quad \text{and} \quad \phi = 60^\circ \text{ in degrees} \]

3(d). What is the RMS value $V_{RMS}$ of $v(t)$?

\[ V_{RMS} = V_m/\sqrt{2} = \frac{25}{\sqrt{2}} = 17.68 \text{ V} \]

\[ V_{RMS} = 17.68 \text{ V} \]
4. Phasor Domain and Source Transformation:
Consider the RLC circuit below. The sinusoidal voltage source is given by \( v_s(t) = 5 \cos(5000t) \).

\[ V_s(t) \rightarrow V_s = 5 \angle 0^\circ \]
\[ C \rightarrow Z_c = \frac{j}{jwC} = \frac{j}{\frac{\sqrt{2}}{5000(4 \times 10^{-6})}} = -j50 \Omega \]
\[ R \rightarrow Z_R = R = 5000 \Omega \]
\[ L \rightarrow Z_L = j \omega L = j(5000 \times 0.01) = j50 \Omega \]

4(a). Convert the circuit to the phasor domain and draw it below.

4(b). Find the total circuit impedance \( Z_{eq} \).

\[
Z_{eq} = Z_c + Z_R \parallel Z_L = -j50 + 5000 \parallel j50
\]
\[
5000 \parallel j50 = \frac{j250000}{5000+j50} \]
\[
5000 \parallel j50 = \frac{j250000(5000 - j50)}{5000 + j50)(5000 - j50)} = \frac{125 \times 10^5 + j125 \times 10^7}{25 \times 10^6 + 2500}
\]
\[
= \frac{125 \times 10^5 + j125 \times 10^7}{25000 + j50} = 0.5 + j49.995 \Omega \approx 0.5 + j50 \Omega
\]

\[
Z_{eq} = Z_c + Z_R \parallel Z_L = -j50 + 0.5 + j50 = 0.5 \Omega
\]

\[
Z_{eq} = 0.5 \Omega
\]
4(c). Does $Z_{eq}$ have any reactive component?  
(circle one): Yes [ ] No [ ] (essentially none)

4(d). Use source transformation to find the phasor voltage $V_2$ across the 10 mH inductor. Also find the corresponding sinusoidal voltage $v_2(t)$. Draw a small sketch of each source transformation that you make.

**Hint:** You can use your result from 4(b) if you only transform the circuit once.

\[
V_s = \frac{V_z}{Z_c} = \frac{5 \angle 0^\circ}{50 \angle -90^\circ} = 1 \angle 90^\circ \ A
\]

Combine $Z_c || Z_L$ into one $Z_P$

\[
Z_P = Z_c || Z_L
\]

\[
\frac{1}{Z_P} = \frac{1}{Z_c} + \frac{1}{Z_L} + \frac{1}{Z_P} = \frac{1}{j \omega C} + \frac{1}{R+j \omega L} + \frac{1}{Z_P} = \frac{1}{j \omega C} + \frac{1}{R} + \frac{1}{j \omega L}
\]

\[
\frac{1}{Z_P} = \frac{1}{j (5000)(4 \times 10^{-4})} + \frac{1}{5000} + \frac{j}{5000 (\omega L)} = \frac{1}{j 20 \times 10^{-3}} + \frac{0.0002}{j \omega L} = \frac{1}{500} \Omega
\]

\[
V_2 = I_s Z_P = (1 \angle 90^\circ ) (5000) = 500 \angle 90^\circ \ V
\]

\[
v_2(t) = 500 \cos(5000t + 90^\circ) \ V
\]
5. Thévenin's Equivalent Circuit in Phasor Domain:
Consider the circuit below. The voltage source $V_s$ and all impedances shown are in phasor form.

5(a). Find the open-circuit phasor voltage $V_{TH}$ across terminals a and b of the Thévenin equivalent circuit.

$$V_{TH} = V_{ab} = I (j100) \text{ V}$$

$$I = \frac{V_s}{Z_{TOT}} = \frac{10\angle 0^\circ}{-j200 + 400 + j100}$$

$$I = \frac{10\angle 0^\circ}{400 - j100} = \frac{10\angle 0^\circ (400 + j100)}{(400 - j100)(400 + j100)} = \frac{4000 + j1000}{160000 + 10000} = \frac{4100 + j1000}{170000}$$

$$I = \frac{4 + j}{170} \text{ A} = \frac{4.123 \angle 14.04^\circ}{170}$$

$$V_{TH} = \left(\frac{4 + j}{170}\right)(j100) = \frac{-100 + j400}{170} = \frac{412.31 \angle 104.14^\circ}{170}$$

$$V_{TH} = 2.425 \angle 104.14^\circ \text{ V}$$

$V_{TH} \approx 4.125 \angle 104.14^\circ \text{ V}$
5(b). Find the Thévenin impedance $Z_{TH}$ of the Thévenin equivalent circuit, by finding the equivalent impedance between terminals a and b.

\[ Z_{TH} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} = \frac{(400 - j200)(j100)}{400 - j200 + j100} = \frac{20000 + j40000}{400 - j100} \]

\[ Z_{TH} = \frac{200 + j400}{4 - j} = \frac{(200 + j400)(2 + j)}{(4 - j)(2 + j)} = \frac{800 - 400 + j1600 + j200}{16 + 1} \]

\[ Z_{TH} = \frac{400 + j1800}{17} = 23.53 + j105.88 \Omega \]

\[ Z_{TH} = 23.53 + j105.88 \Omega = 108.47 \angle 77.47^\circ \Omega \]

6. Maximum Power Transfer:
6(a). Using the circuit in the previous problem 6, what value of load impedance $Z_L$ maximizes the average (real) power transferred to the load $Z_L$? You don’t have to prove what value will provide maximum power, just use the appropriate value of $Z_L$ that does provide maximum power.

\[ Z_L = Z_{TH}^* = 23.53 - j105.88 \Omega = 108.47 \angle -77.47^\circ \Omega \]

\[ Z_L = 23.53 - j105.88 \Omega = 108.47 \angle -77.47^\circ \Omega \]

6(b). Using the value of $Z_L$ obtained above, find the average (real) power $P_L$ absorbed by the load $Z_L$.

\[ P_L = \left| V_{TH_m} \right|^2 / 8R_L = (2.425)^2 / (8 \times 25.53) \text{ W} \]

\[ P_L = 0.0288 \text{ W} \]

\[ P_L = 28.8 \text{ mW} \]
7. Power Factor and Average, Reactive, Complex and Apparent Power:
Consider the circuit below.

\[ V = 50 \angle 0^\circ \]
\[ Z_L = 100 + j173.2 \Omega \]
\[ I = \frac{V}{Z} = \frac{50 \angle 0^\circ}{200 \angle 60^\circ} = 0.25 \angle -60^\circ \]
\[ = 300 \angle 60^\circ \Omega \]

7(a). Find the average power \( P \) absorbed by the load impedance \( Z_L \):
\[ P = \frac{1}{2} I_m^2 R = \frac{(0.25)^2}{2} (100) = \frac{(4)^2 (100)}{2} = \frac{100}{32} = 3.125 \text{ W} \]
\[ P = 3.125 \text{ W} \]

7(b). Find the reactive power \( Q \) absorbed by the load impedance \( Z_L \):
\[ Q = \frac{1}{2} I_m^2 X = \frac{(0.25)^2}{2} (173.2) = 5.4125 \text{ VAR} \]
\[ Q = 5.4125 \text{ VAR} \]

7(c). Find the complex power \( S \) absorbed by the load impedance \( Z_L \), and the apparent power \( |S| \):
\[ S = P + jQ \]
\[ |S| = \sqrt{P^2 + Q^2} \]
\[ S = 3.125 + j5.4125 \]
\[ |S| = 6.25 \]

7(d). Compute the power factor for the load impedance \( Z_L \):
\[ pf = \cos (\theta_V - \theta_l) = \cos (60^\circ) = 0.5 \]
\[ pf = 0.5 \]

7(e). Is the power factor leading or lagging? (circle one): leading \[ \square \] lagging \[ \square \]