BIO 682
Nonparametric Statistics
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http://www4.nau.edu/shustercourses/BIO682/index.htm

Lecture 8

Example: Sign Test

1. The number of warning cries delivered against intruders by male and female pairs of trogons.

Method:

For N = 10, count the *smallest number* of + or – (here = 2).

a. Don't count ties or 0.

Pair	M	F	direction	Sign
1	8	3	>	+
2	4	3	>	+
3	6	4	>	+
4	6	3	>	+
5	3	3	=	0
6	2	6	<	-
7	5	2	>	+
8	3	3	=	0
9	1	2	<	-
10	6	0	>	+

For N < 35

1. Look up P in a table of binomial probabilities.
2. In this case, P=.055 (1-tailed test).
3. But if no direction is predicted, use 2P (=.110, ns).

TABLE D. TABLE OF PROBABILITIES ASSOCIATED WITH VALUES AS SMALL AS OBSERVED VALUES OF x IN THE BINOMIAL TEST*

Given in the body of this table are one-tailed probabilities under H_0 for the binomial test when $P = Q = \frac{1}{2}$. To save space, decimal points are omitted in the p's.

N \ x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5	031	188	500	812	969	†										
6	016	109	344	656	891	984	†									
7	008	062	227	500	773	938	992	†								
8	004	035	145	363	637	855	965	996	†							
9	002	020	090	254	500	746	910	980	998	†						
10	001	011	055	172	377	623	828	945	989	999	†					
11		006	033	113	274	500	726	887	967	994	†	†				
12		003	019	073	194	387	613	806	927	981	997	†	†			
13		002	011	046	133	291	500	709	867	954	989	998	†	†		
14		001	006	029	090	212	395	605	788	910	971	994	999	†	†	
15			004	018	059	151	304	500	696	849	941	982	996	†	†	†
16			002	011	038	105	227	402	598	773	895	962	989	998	†	†
17			001	006	025	072	166	315	500	685	834	928	975	994	999	†
18			001	004	015	048	119	240	407	593	760	881	952	985	996	999
19				002	010	032	084	180	324	500	676	820	916	968	990	998
20				001	006	021	058	132	252	412	588	748	868	942	979	994
21				001	004	013	039	095	192	332	500	668	808	905	961	987
22					002	008	026	067	143	262	416	584	738	857	933	974
23					001	005	017	047	105	202	339	500	661	798	895	953
24					001	003	011	032	076	154	271	419	581	729	846	924
25						002	007	022	054	115	212	345	500	655	788	885

* Adapted from Table IV, B, of Walker, Helen, and Lev, J. 1953. *Statistical inference.* New York: Holt, p. 458, with the kind permission of the authors and publisher.

† 1.0 or approximately 1.0.

Wilcoxon Signed-Ranks Test

1. A more powerful test because it provides a test of:
   a. Order (like sign test)
   b. magnitude.
2. Useful in behavioral tests when information is known about magnitude:
   a. It is then possible to rank pairs based on *magnitude of differences*.

Wilcoxon Signed-Ranks Test: Method

1. Use the same table as before, except add the following:
   a. Calculate unsigned difference between M and F.
   b. Rank them.
   c. Add the sign of the difference to the rank.
2. Note, 0 is omitted from ranks.

Wilcoxon Signed-Ranks Test: Method

Pair	M	F	dir.	d	rank	signed rank
1	8	3	>	5	7	7
2	4	3	>	1	1.5	1.5
3	6	4	>	2	3	3
4	6	3	>	3	4.5	4.5
5	3	3	=	0	–	
6	2	6	<	4	6	-6
7	5	2	>	3	4.5	4.5
8	3	3	=	0	–	
9	1	2	<	1	1.5	-1.5
10	6	0	>	6	8	8

Wilcoxon Signed-Ranks Test: Method

2. Determine N (# of non-zero d's = 8)

3. Calculate:

a. T^+, sum of (+) ranks (= 28.5).

b. T^-, sum of (-) ranks (= -7.5).

4. The smaller absolute value is T_s, look up on Table 30.

a. Here, T_s = 7.5, > 6 for which P = 0.0547, ns.

TABLE 30 Critical values of the Wilcoxon rank sum.

Nominal α	0.05		0.025		0.01		0.005	
n	T	α	T	α	T	α	T	α
5	0	.0312						
	1	.0625						
6	2	.0469	0	.0156				
	3	.0781	1	.0312				
7	3	.0391	2	.0234	0	.0078		
	4	.0547	3	.0391	1	.0156		
8	5	.0391	3	.0195	1	.0078	0	.0039
	6	.0547	4	.0273	2	.0117	1	.0078
9	8	.0488	5	.0195	3	.0098	1	.0039
	9	.0645	6	.0273	4	.0137	2	.0059
10	10	.0420	8	.0244	5	.0098	3	.0049
	11	.0527	9	.0322	6	.0137	4	.0068
11	13	.0415	10	.0210	7	.0093	5	.0049
	14	.0508	11	.0269	8	.0122	6	.0068
12	17	.0461	13	.0212	9	.0081	7	.0046
	18	.0549	14	.0261	10	.0105	8	.0061
13	21	.0471	17	.0239	12	.0085	9	.0040
	22	.0549	18	.0287	13	.0107	10	.0052
14	25	.0453	21	.0247	15	.0083	12	.0043
	26	.0520	22	.0290	16	.0101	13	.0054
15	30	.0473	25	.0240	19	.0090	15	.0042
	31	.0535	26	.0277	20	.0108	16	.0051
16	35	.0467	29	.0222	23	.0091	19	.0046
	36	.0523	30	.0253	24	.0107	20	.0055
17	41	.0492	34	.0224	27	.0087	23	.0047
	42	.0544	35	.0253	28	.0101	24	.0055
18	47	.0494	40	.0241	32	.0091	27	.0045
	48	.0542	41	.0269	33	.0104	28	.0052
19	53	.0478	46	.0247	37	.0090	32	.0047
	54	.0521	47	.0273	38	.0102	33	.0054
20	60	.0487	52	.0242	43	.0096	37	.0047
	61	.0527	53	.0266	44	.0107	38	.0053

TABLE H
Critical values of T^+ for the Wilcoxon signed ranks test

Table entries for a given N is $P[T^+ \geq c]$, the probability that T^+ is greater than or equal to the sum c.

	N						
c	3	4	5	6	7	8	9
3	.6250						
4	.3750						
5	.2500	.5625					
6	.1250	.4375					
7		.3125					
8		.1875	.5000				
9		.1250	.4063				
10		.0625	.3125				
11			.2188	.5000			
12			.1563	.4219			
13			.0938	.3438			
14			.0625	.2813	.5313		
15			.0313	.2188	.4688		
16				.1563	.4063		
17				.1094	.3438		
18				.0781	.2891	.5273	
19				.0469	.2344	.4727	
20				.0313	.1875	.4219	
21				.0156	.1484	.3711	
22					.1094	.3203	
23					.0781	.2734	.5000
24					.0547	.2305	.4551
25					.0391	.1914	.4102
26					.0234	.1563	.3672
27					.0156	.1250	.3262
28					.0078	.0977	.2852
29						.0742	.2480
30						.0547	.2129
31						.0391	.1797
32						.0273	.1504
33						.0195	.1250
34						.0117	.1016
35						.0078	.0820
36						.0039	.0645
37							.0488
38							.0371
39							.0273
40							.0195
41							.0137
42							.0098
43							.0059
44							.0039
45							.0020

For Large Samples,

1. As before, the values of T^+ begin to approximate a normal distribution.

$$z = \frac{T^+ - \mu_{T+}}{\sigma_{T+}}$$

Where: $\mu_{T+} = N(N+1)/4$

and $\sigma_{T+} = N(N+1)(2N+1)/24$

Two Sample Tests

1. U-test, Wilcoxon's test

a. Tests used to determine if two independent samples have same median.

b. Analogous to *t*-test

c. Use ordinal data, which *t*-test will not accommodate (very well).

t-test or U-test?

a. If data meet assumptions of *t*-test, use it, smaller N needed than U- or W-tests.

b. However, if sample size can be increased, can be as/more powerful than t-test

U-test, Wilcoxon's test

1. Both tests devised at about the same time.

a. Method is slightly different, but same results are obtained.

2. Wilcoxon also produces the same statistic as a paired t-test.

a. Should be used in the same situations.

U-test: Example

Thermosphaeroma isopods, inhabit hot spring in New Mexico.

1. Small environment, isopods keep pool free of predators.

2. Sexual dimorphism is great, males guard females.

a. Female sexual receptivity associated with molt.

b. Females are spatially dispersed, apparent time constraints on male guarding time.

Male Mate Discrimination

1. Males observed assessing females.

a. Females held > 5 sec, < 15 min -were rejects.

b. Females held over 15 min – were paired.

U-Test: Method

MANN-WHITNEY UTEST OF THERMOSPHAEROMA DATA

PAIRED	BLENG	RANK	UNPAIRED	BLENG	RANK
1	30	47	2	24.5	28.5
1	28	37	2	18	14.5
1	18	14.5	2	28	37
1	38.25	57.5	2	18	14.5
1	30	47	2	28	37
1	36	54.5	2	18	14.5
1	18	14.5	2	24.5	28.5
1	36	54.5	2	18	14.5
1	19.5	20	2	28	37
1	16.5	7	2	18	14.5
1	30	47	2	28	37
1	30	47	2	18	14.5
1	22.75	23.5	2	22.75	23.5
1	18	14.5	2	16.5	7
1	28	37	2	28	37
1	18	14.5	2	32	51.5
1	22.75	23.5	2	28	37
1	36	54.5	2	16.5	7
1	22.75	23.5	2	24.5	28.5
1	32	51.5	2	16.5	7
1	28	37	2	24.5	28.5
1	30	47	2	16.5	7
1	28	37	2	22.75	23.5
1	38.25	57.5	2	15	3.5
1	28	37	2	28	37
1	30	47	2	12.25	1.5
1	36	54.5	2	22.75	23.5
1	30	47	2	15	3.5
1	28	37	2	12.75	1.5
29	27.81896	1091	29	21.49137	620
	6.422917			5.467175	

1. Rank body sizes of paired (n_1), and rejected females (n_2):
   a. As a single series.
   b. Lowest to highest.
   c. Calculate R_1, R_2 as Σr_{ij}.

U-Test: Method

2. If N < 20, Calculate U as:

$$n_1 n_2 + n_2(n_2 + 1)/2 - R_2$$

3. if N > 20, calculate z as approximation of normal dist:

$$z = \frac{U - \mu_U}{\sigma_U}$$

where: $\mu_U = n_1 n_2 / 2$

$$\sigma_U = \sqrt{[n_1 n_2 (n_1 + n_2 + 1)/12]}$$

Tied Ranks:

1. Changes the variability in the set of ranks
   a. Therefore, need to adjust the value of σ_U

$$\sigma_U = \sqrt{[n_1 n_2 / N(N-1)][(N^3 - N)/12 - \Sigma T]}$$

where

$$\Sigma T = \sum^{t} (t^3 - t)/12$$

   b. t = number of tied scores for a given rank
   c. ΣT amounts to a term in the denominator that accounts for the length of the run of tied scores.

Note That,

c. This correction makes the value of z *larger*

d. Thus, the uncorrected test is more *conservative*.

$$z = \frac{U - \mu_U}{\sigma_U}$$

$$\sigma_U = \sqrt{[n_1 n_2/N(N-1)][(N^3 - N)/12 - \Sigma T]}$$

k-Sample Cases

1. Friedman's 2-way ANOVA

a. A test to determine whether k (= a) matched samples are from the same population.

b. Example: testing four groups of a individuals with four different feeding regimes.

c. See S&C for details on this one.

d. This test *must* have a balanced design (all samples equal) and be fully factorial (all cells filled) to work.

k-Sample Cases

2. Kruskal-Wallis test

a. A non-parametric ANOVA

1. Tests the hypothesis that the *medians* of a groups are equal.

2. Data cast in a 2 way table.

Group				
1	2	3	...	k
x_{11}	x_{12}	x_{13}	...	x_{1a}
x_{21}	x_{22}	x_{23}	...	x_{2a}
.				
x_{n1}				x_{na}

Kruskal-Wallis Test

3. The N observations are replaced by ranks in a single series.
   a. Ranking is in order, low to high.
   b. Tied ranks as before (1+2+3)/3 = 2

White	Yellow	Purple
96	82	115
128	124	149
83	132	166
61	135	147
101	109	

	White	Yellow	Purple
	4	2	7
	9	8	13
	3	10	14
	1	11	12
	5	6	
R_j	22	37	46

Kruskal-Wallis H

1. Ranks are then summed for each of the a groups, where,

$R_j = (\Sigma r_{ij})$ = the sum of i ranks for each j-th column.

n_j = number of cases in the j-th column

N = total number of cases.

$$H = \frac{12}{N(N+1)} \sum^{a} \frac{R_j^2}{n_j} - 3(N+1)$$

with df = a-1

Kruskal-Wallis: Example

1. The numbers of beetles on three colors of flowers

White	Yellow	Purple
96	82	115
128	124	149
83	132	166
61	135	147
101	109	

Kruskal-Wallis: Example

2. Rank the scores as a *single series* from lowest to highest.

	White	Yellow	Purple
	4	2	7
	9	8	13
	3	10	14
	1	11	12
	5	6	
R_j	22	37	46

Kruskal-Wallis: Example

	White	Yellow	Purple
	4	2	7
	9	8	13
	3	10	14
	1	11	12
	5	6	
R_j	22	37	46

3. Then use K-W formula to calculate H:

$$H = \frac{12}{N(N+1)} \sum^{a} \frac{R_j^2}{n_j} - 3(N+1)$$

with df = a-1

$$= \frac{12}{14(14+1)} [(22)^2/5 + (37)^2/5 + (46)^2/4] - 3(14+1)$$

= 6.4, df= 2

TABLE O
Critical values for the Kruskal-Wallis one-way analysis of variance by ranks statistic, *KW*

Sample sizes			α				
n_1	n_2	n_3	.10	.05	.01	.005	.001
2	2	2	4.25				
3	2	1	4.29				
3	2	2	4.71	4.71			
3	3	1	4.57	5.14			
3	3	2	4.56	5.36			
3	3	3	4.62	5.60	7.20	7.20	
4	2	1	4.50				
4	2	2	4.46	5.33			
4	3	1	4.06	5.21			
4	3	2	4.51	5.44	6.44	7.00	
4	3	3	4.71	5.73	6.75	7.32	8.02
4	4	1	4.17	4.97	6.67		
4	4	2	4.55	5.45	7.04	7.28	
4	4	3	4.55	5.60	7.14	7.59	8.32
4	4	4	4.65	5.69	7.66	8.00	8.65
5	2	1	4.20	5.00			
5	2	2	4.36	5.16	6.53		
5	3	1	4.02	4.96			
5	3	2	4.65	5.25	6.82	7.18	
5	3	3	4.53	5.65	7.08	7.51	8.24
5	4	1	3.99	4.99	6.95	7.36	
5	4	2	4.54	5.27	7.12	7.57	8.11
5	4	3	4.55	5.63	7.44	7.91	8.50
5	4	4	4.62	5.62	7.76	8.14	9.00
5	5	1	4.11	5.13	7.31	7.75	
5	5	2	4.62	5.34	7.27	8.13	8.68
5	5	3	4.54	5.71	7.54	8.24	9.06
5	5	4	4.53	5.64	7.77	8.37	9.32
5	5	5	4.56	5.78	7.98	8.72	9.68
Large samples			4.61	5.99	9.21	10.60	13.82

Note: The absence of an entry in the extreme tails indicates that the distribution may not take on the necessary extreme values.

Adapted from Table F in Kraft, C. H., and van Eeden, C. (1968). A nonparametric introduction to statistics. New York: Macmillan, with the permission of the publisher.

Kruskal-Wallis: Significance

1. Note that the smallest n_j is < 5:

a. For $a > 3$ and $n_j > 5$, H is distributed as X^2 with df = a-1.

b. For smaller values of n_j, must use exact probability table.

2. See Table O:

a. Match up sample sizes.

b. Find value of H and read off exact probability ($H_{obs} = 6.4$).

$H_{[.05;5,5,4]} = 5.64$, P<0.05.

Kruskal-Wallis: Tied Ranks

1. To correct for large numbers of tied scores,

a. Calculate D (S&R):

$$D = 1 - \frac{\Sigma T}{N^3 - N}$$

t

Where: $\Sigma T = \Sigma(t^3 - t)/12$, and t = number of tied scores for a given rank.